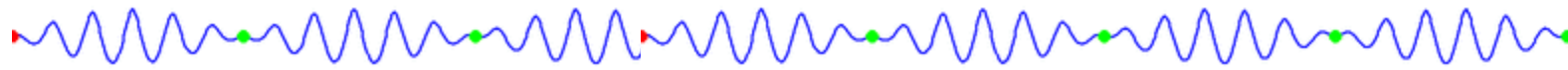


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QHP5701 Exploratory Data Analysis

Signals

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Contents

- Overview of Signals & Systems
- What are Signals?

QHP5701 Exploratory Data Analysis

Overview of Signals & Systems

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Overview of Signals & Systems

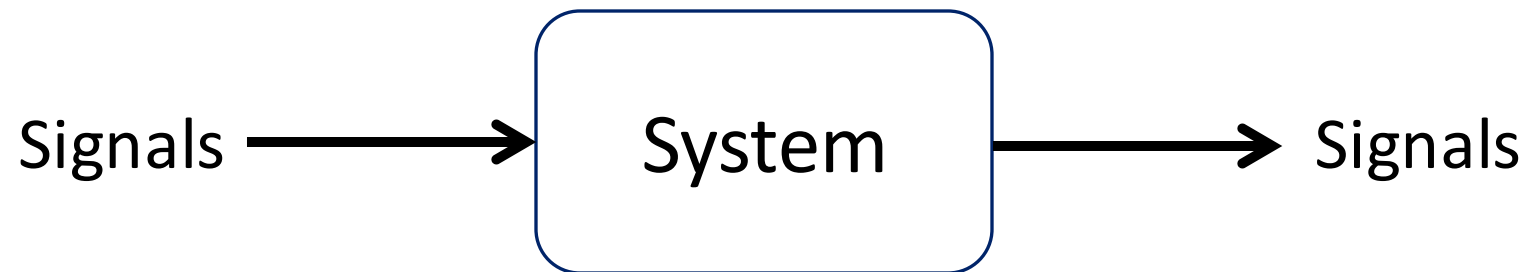
- Signals and Systems?

Any activity of human, can be thought of interaction or an interplay of the signals & systems

- A system as a block:

- **is a meaningful** interconnection of physical devices or components
- is an interconnection of subsystems, which are composed of physical devices or components

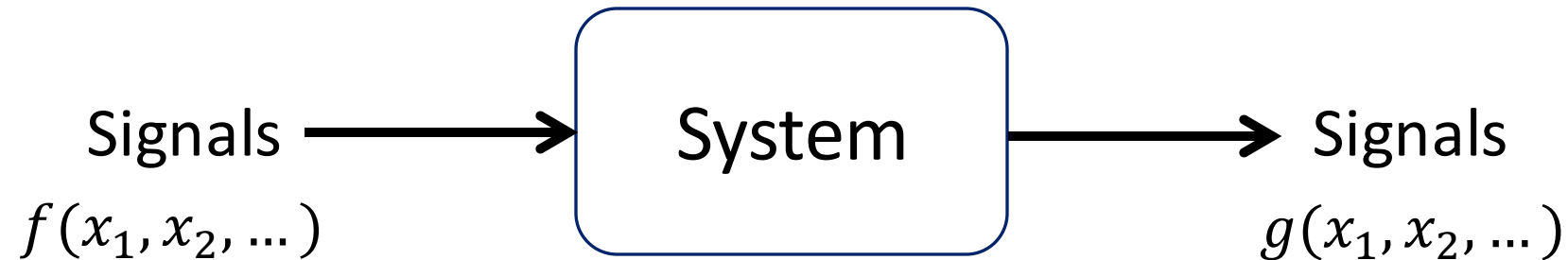
- A system by itself cannot achieve anything, it must be closely related to signals



Overview of Signals & Systems

- Signals and Systems?

Any activity of human, can be thought of interaction or an interplay of the signals & systems



*A **signal**, in general, is a function of one or more independent variables*

System takes a signal, e.g. $x(t)$, as input and produced more desirable output $y(t)$

e.g. Signal processing blocks

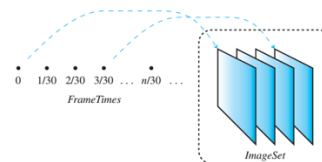
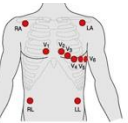
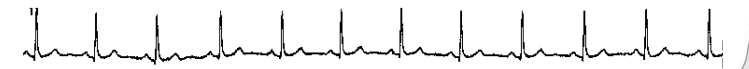
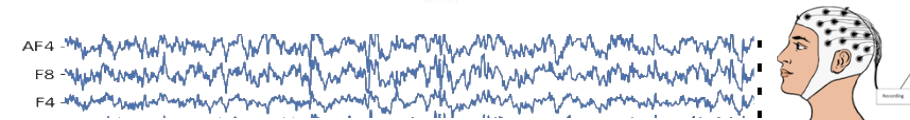
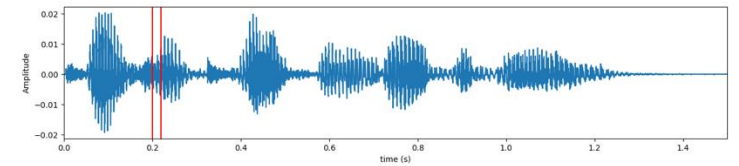
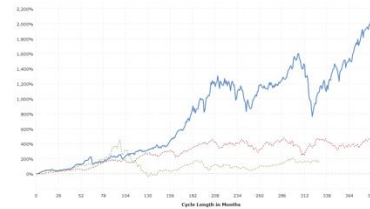
Signals

A **signal**, in general, is a function of one or more independent variables

$$f(x_1, x_2, \dots), x(t)$$

Examples

- Temperature in Hainan: $x(t)$
- Speech/audio: $x(t)$
- EEG, ECG, $x(t)$
- Image $I(x,y)$
- Video $V(x,y,t)$



Magnetic Resonance Image (MRI) data as 2-dimensional signal



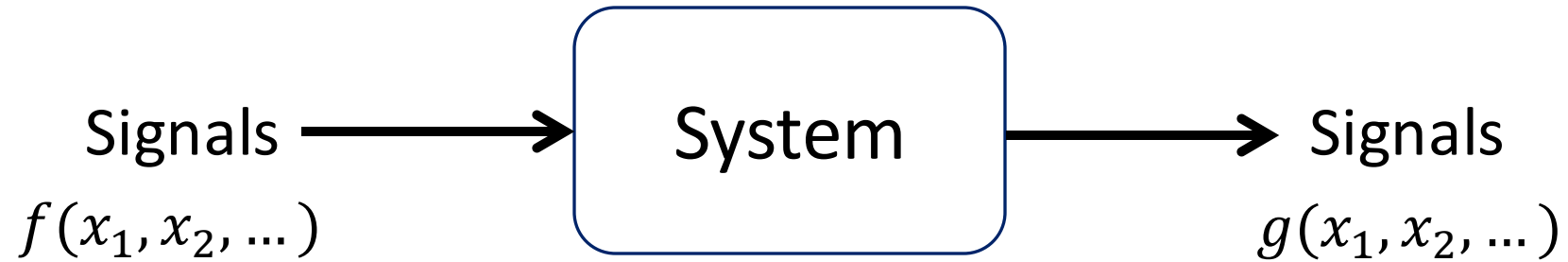
Signals

A **signal**, in general, is a function of one or more independent variables

$$f(x_1, x_2, \dots), \quad x(t)$$

Anything that changes over one or more dimensions can be thought as a signal

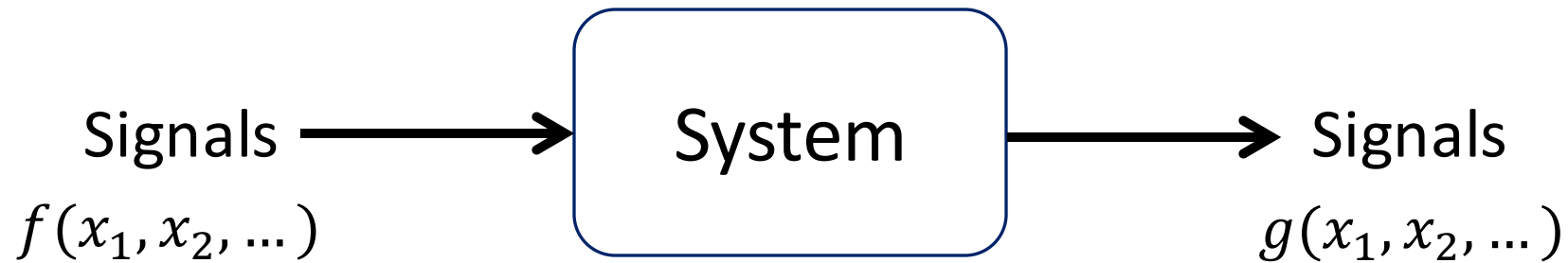
Examples of Signals & Systems



Examples

- Electrical Circuit
- Motor
- Heart Monitor
- ...
- ...

Signals & Systems: Problems



Analysis Problems

- Given: Input signal and system Find: output signal

Design/Synthesis Problems

- Given: Input signal and desired output signal Design: System

Signals & Systems: Why?

- Foundation of many engineering fields
- Provides tools to analyse signals and systems
- Used in variety of other fields
- Laydown foundations for systems design

Note: Most of slides will be empty for in-class computations

Since we are following a text-book heavily, slides, will not include all the mathematical computations and details. Please check the text-book, for details.

QHP5701 Exploratory Data Analysis

Signals

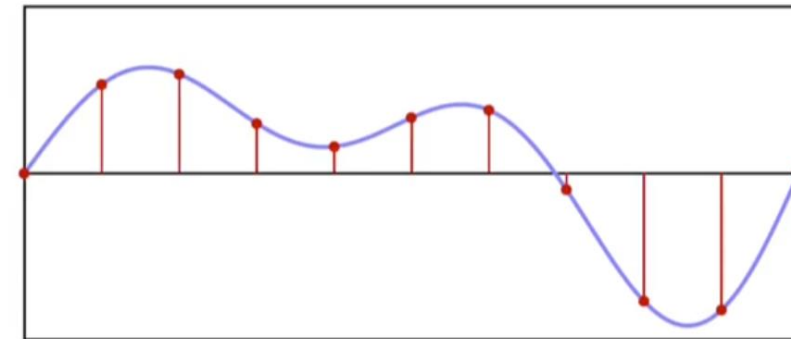
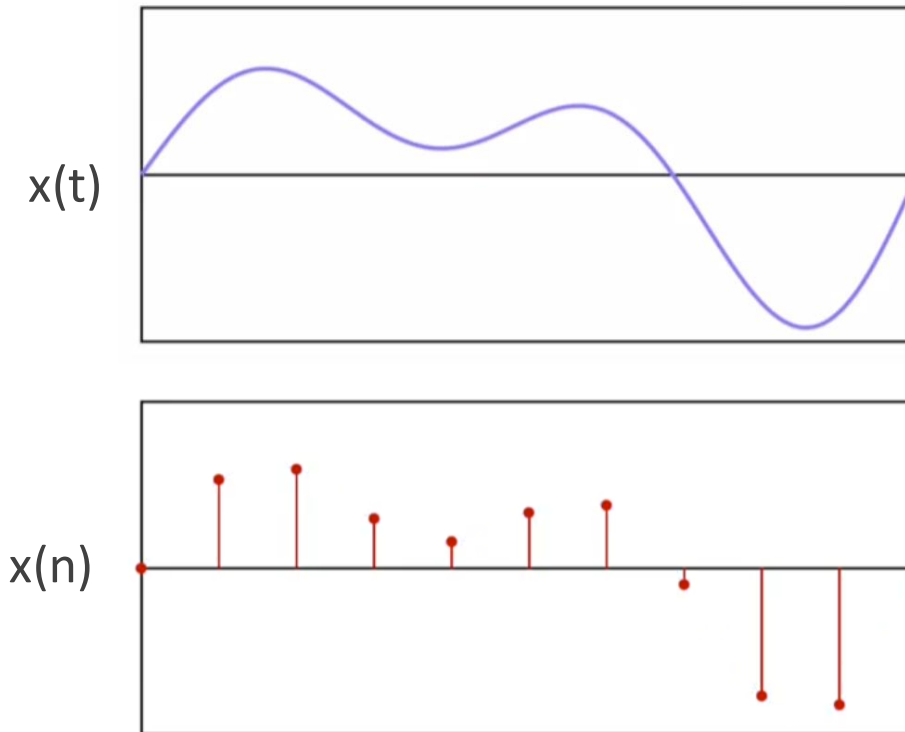
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Signals:

- $x(t)$: continues-time signal
- $x(n)$: discreet-time signal

Reconstruction of Signal

- $x(t)$: continues-time signal
- $x(n)$: discreet-time signal



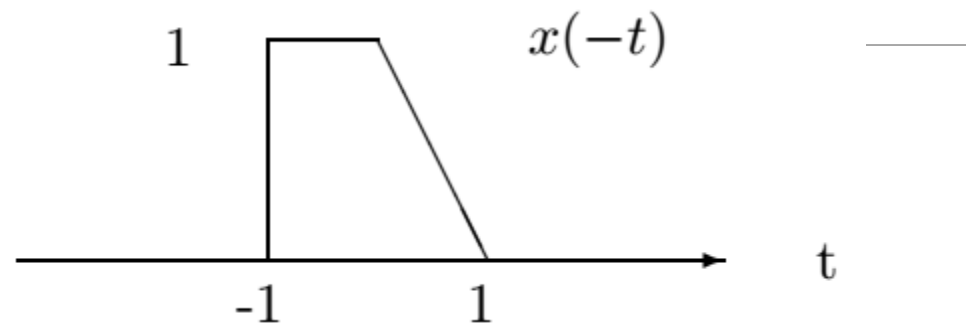
Signals: Transformation

- **Folding**, time-reversal: $x(-t)$, $x(-n)$
- **Shifting**, delayed, advanced: $x(t \pm T)$, $x(n \pm N)$
- **Scaling**, compression and expansion: $x(\alpha t)$, $x(kn)$
- **Amplitude Scaling**, amplification and attenuation: $Ax(t)$

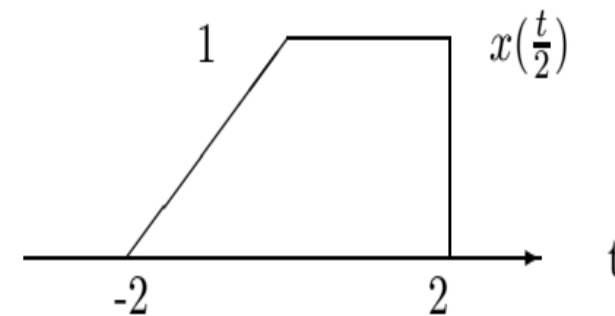
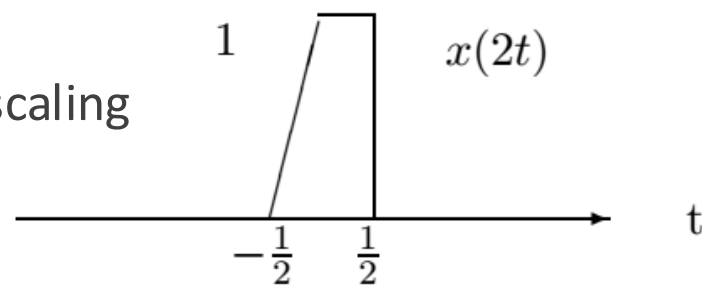
Computation

Examples

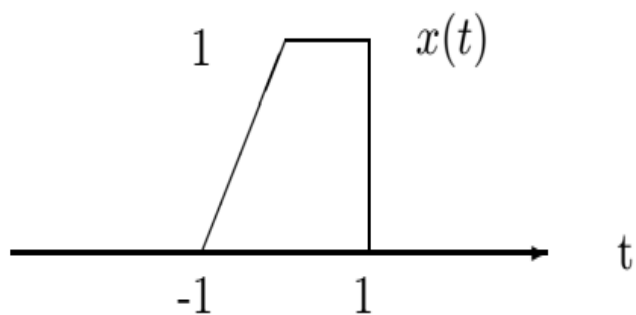
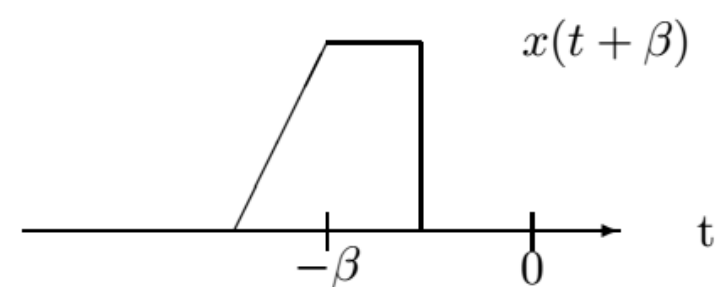
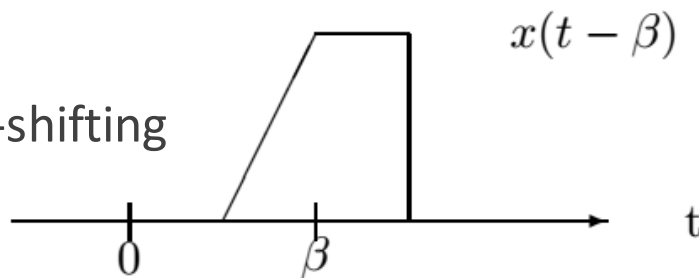
time-reversal



time-scaling



time-shifting



Note: Most of slides will be empty for in-class computations

Since we are following a text-book heavily, slides, will not include all the mathematical computations and details. Please check the text-book, for details.

QHP5701 Exploratory Data Analysis

Signals: Classification of signals

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Signals: Classification

- Classification of Signals: Properties (attributes/features) of signals
 - Continues & Discrete
 - Analog & Digital
 - Periodic & Aperiodic
 - Even & Odd
 - Energy & Power
 - Real & Complex
 - Deterministic & Random & Chaotic signal

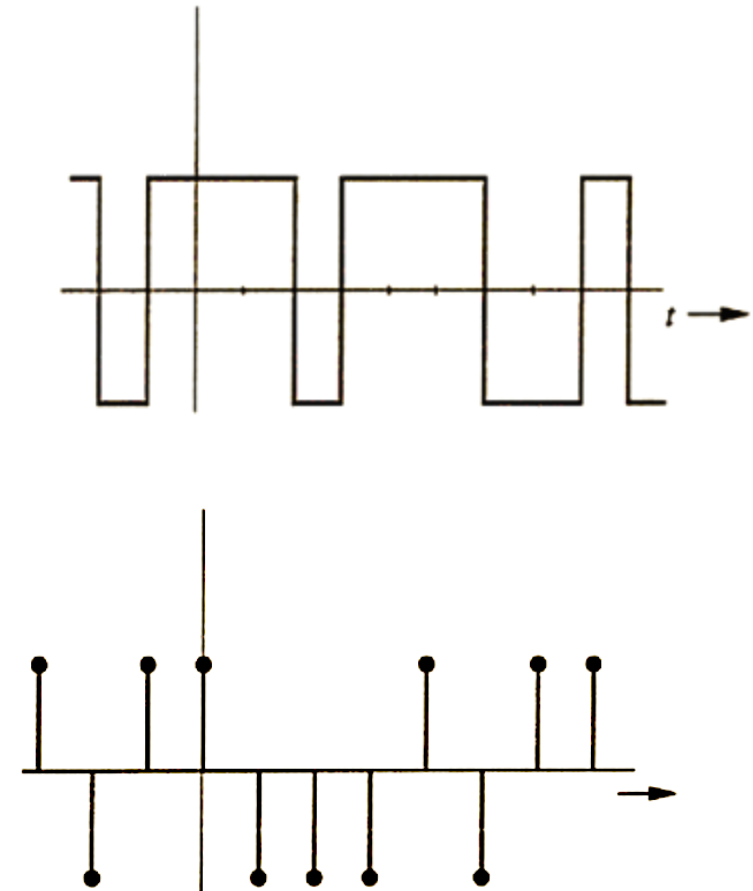
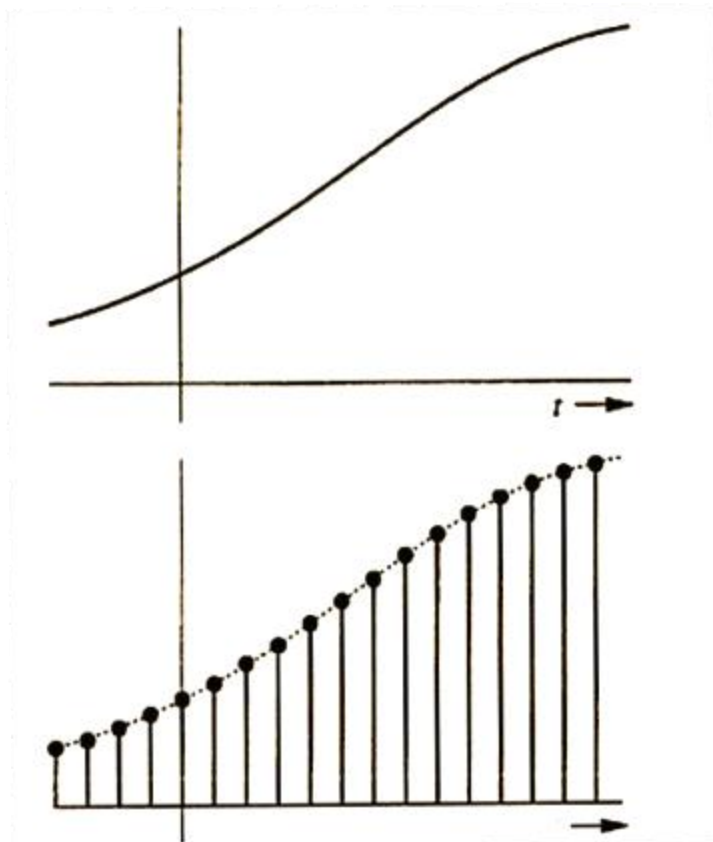
Continues & Discrete time signals

Analog & Digital signals

Cont. & Discrete | Analog & Digital

Which one is which?

- Digital, CT
- Analog, CT
- Digital, DT
- Analog, DT



Periodic & Aperiodic

$$\text{Periodicity: } x(t) = x(t + T)$$

Periodic & Aperiodic

Periodicity: $x(t) = x(t + T)$

$$x(t) = x(t + mT)$$

$$x(t) = x(t + \alpha)$$

fundamental period = $T_o = \min\{\alpha\}$

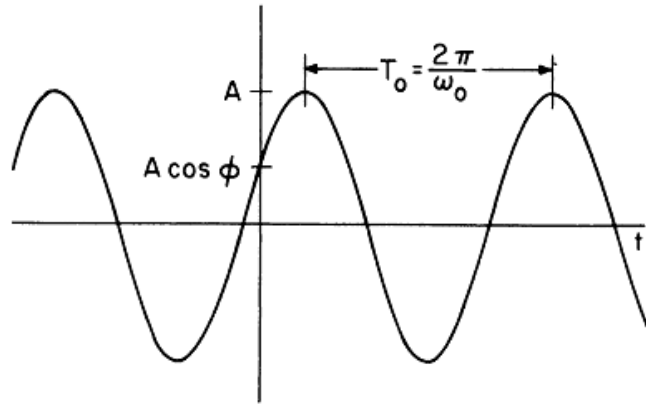
fundamental frequency = $f = \frac{1}{T_o}$ cycles/s (Hz)

$$x(n) = x(n + N)$$

Periodic & Aperiodic

Periodicity: $x(t) = x(t + T)$

$$x(t) = A \cos(\omega_0 t + \phi)$$



Periodic & Aperiodic

Periodicity: $x(t) = x(t + T)$

$$x(t) = \cos(\omega_0 t)$$

$$x(t) = \cos(2\pi t)$$

$$x(t) = \cos(t/6)$$

Periodic & Aperiodic

$$\text{Periodicity: } x(t) = x(t + T)$$

Periodic & Aperiodic

Periodicity: $x(n) = x(n + N)$

$$x(n) = \cos(\Omega_o n)$$

$$x(n) = \cos(2\pi n)$$

$$x(n) = \cos(n/6)$$

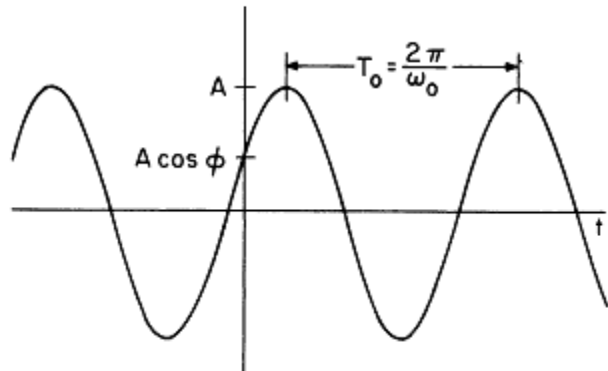
Periodic & Aperiodic

$$\text{Periodicity: } x(n) = x(n + N)$$

Periodicity: Examples

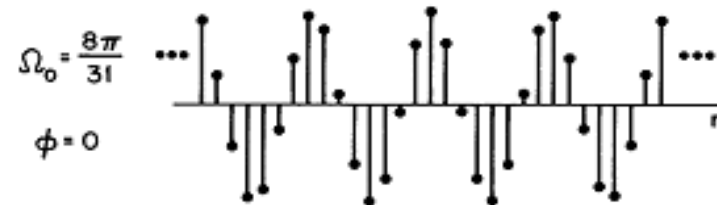
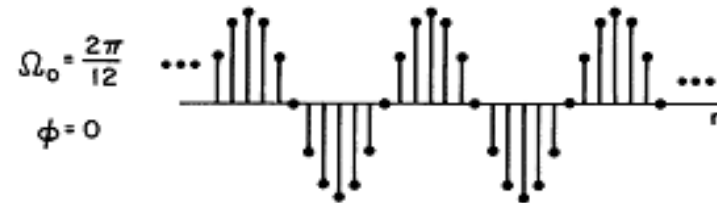
$$T_o = \frac{2\pi m}{\omega_o} \Rightarrow \text{period} = \frac{2\pi}{\omega_o}$$

$$x(t) = A \cos(\omega_o t + \phi)$$

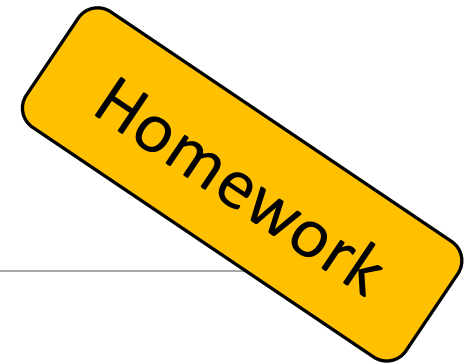


$$\Omega_o = \frac{2\pi m}{N}$$

$$x[n] = A \cos(\Omega_o n + \phi)$$



Periodicity: Deductions!!



Deduce following with possible answers as:

Always Periodic, Not Always Periodic, Always Aperiodic, Not Always Aperiodic

Answers will be different for Continues Time and Discrete Time Signals So answer separately for Continues and Discrete

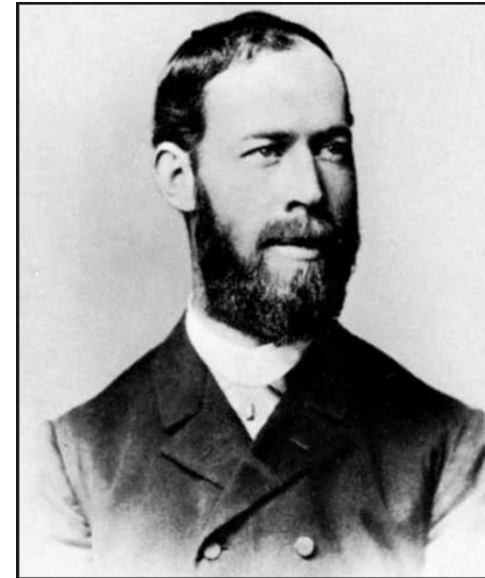
Cases	Continues	Discrete	
Periodic + Periodic =	_____	_____	(+ and -)
Periodic + Aperiodic =	_____	_____	
Aperiodic + Aperiodic =	_____	_____	
Periodic x Periodic =	_____	_____	
Periodic x Aperiodic =	_____	_____	
Aperiodic x Aperiodic =	_____	_____	

Heinrich Hertz

Measure for frequency

cycles/s = Hertz = Hz

- German physicist, most of work on radio waves and electromagnetism
- Died very young, at age of 37



Heinrich Rudolf Hertz
German physicist
(1857-1894)

Even & Odd

Even signal: $x(t) = x(-t)$

Even & Odd

Odd signal: $x(t) = -x(-t)$

Even & Odd

$$x(t) = \text{Ev}\{x(t)\} + \text{Od}\{x(t)\}$$

$$\text{Ev}\{x(t)\} = \frac{x(t) + x(-t)}{2}$$

$$\text{Od}\{x(t)\} = \frac{x(t) - x(-t)}{2}$$

Energy & Power

- Instantaneous power $p(t)$

- Total energy over duration $E = \int_{t_1}^{t_2} |x(t)|^2 dt$

- Average power over duration

- Total energy $E_{\infty} = \int_{-\infty}^{\infty} |x(t)|^2 dt$

- Total power

$$E = \sum_{n=n_1}^{n_2} |x(n)|^2$$

Energy & Power

Energy & Power

Energy & Power Signal

Energy Signal

- A signal with finite energy; $E < \infty$, is called energy signal
- it has 0 average power, $P=0$

Power Signal

- A signal with infinite energy $E=\infty$ and finite average power $P<\infty$ are called power signal

Neither Energy nor Power signal

- A signal with infinite energy and Infinite power falls in this category
- $E = \infty$ & $P = \infty$

Real & Complex signal

Deterministic & Random & Chaotic signal

QHP5701 Exploratory Data Analysis

Signals: Elementary signals

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Signals: Elementary

- Unit Step
- Unit Impulse function, Direc Delta
- Generalised Exponential
 - Exponential
 - Sinusoidal
 - Complex Exponential
- Unit Ramp
- Gate function – Square
- Signum function

Test for:

- Periodicity
- Even Odd
- Energy, Power

Test for:

- Periodicity
- Even Odd
- Energy, Power

Signals: Elementary

Unit step $u(t)$, $u(n)$

Test for:

- Periodicity
- Even Odd
- Energy, Power

Signals: Elementary

Unit step $u(t)$, $u(n)$

Signals: Elementary

Unit Impulse function $\delta(t), \delta(n)$

Test for:

- Periodicity
- Even Odd
- Energy, Power

- Test for:
- Periodicity
 - Even Odd
 - Energy, Power

Signals: Elementary

- Practicality of $u(t)$ and $\delta(t)$
- Relation of $u(t)$ and $\delta(t)$ also in DT

- Test for:
- Periodicity
 - Even Odd
 - Energy, Power

Signals: Elementary

- Use of delta $x(t)\delta(t)$
- Sampling/Sifting property of delta

Signals: Elementary

Generalised Exponential function

- Exponential
- Complex Exponential
- Sinusoidal

$$x(t) = Ce^{at}$$

C = complex or real
a = complex or real

Test for:

- Periodicity
- Even Odd
- Energy, Power

Signals: Elementary

Generalised Exponential function $x(t) = Ce^{at}$

Signals: Elementary

Generalised Exponential function $x(t) = Ce^{at}$

Signals: Elementary

Generalised Exponential function $x(t) = Ce^{at}$

Signals: Elementary

Generalised Exponential function

$$x(n) = C\alpha^n = Ce^{\beta n}$$

C = complex or real
 α = complex or real

Test for:

- Periodicity
- Even Odd
- Energy, Power

Signals: Elementary

Generalised Exponential function

$$x(n) = C\alpha^n = Ce^{\beta n}$$

C = complex or real
 α = complex or real

Signals: Elementary

Generalised Exponential function $x(n) = C\alpha^n = Ce^{\beta n}$

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Signals: Elementary

Generalised Exponential function $x(n) = C\alpha^n = Ce^{\beta n}$

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Signals: Elementary

Generalised Exponential function

$$x(n) = C\alpha^n = Ce^{\beta n}$$

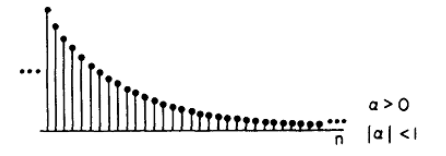
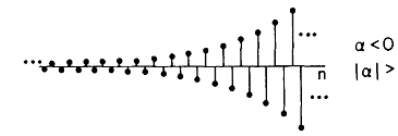
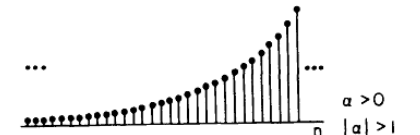
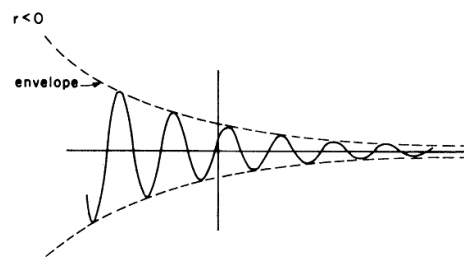
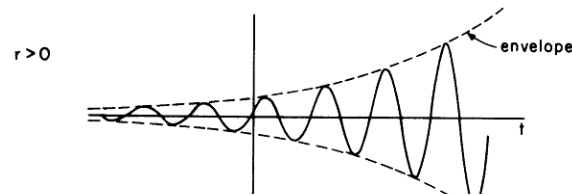
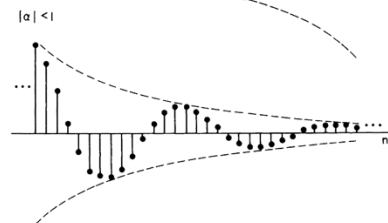
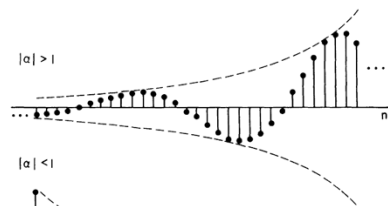
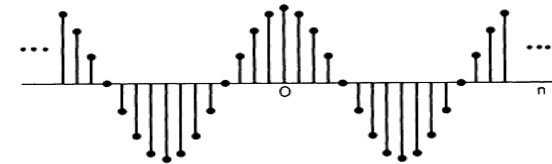
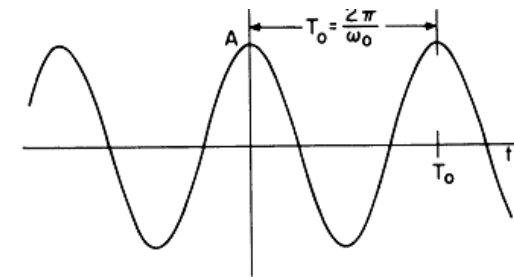
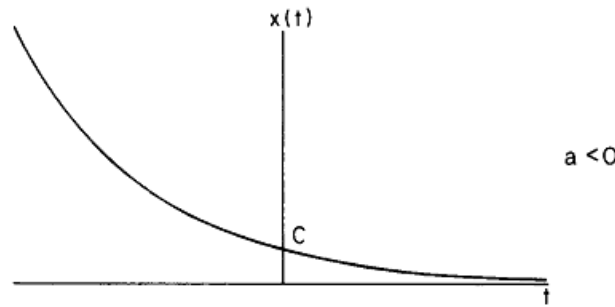
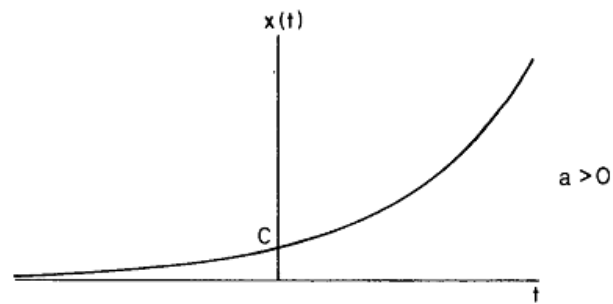
C = complex or real
 α = complex or real

Signals: Elementary

Generalised Exponential function

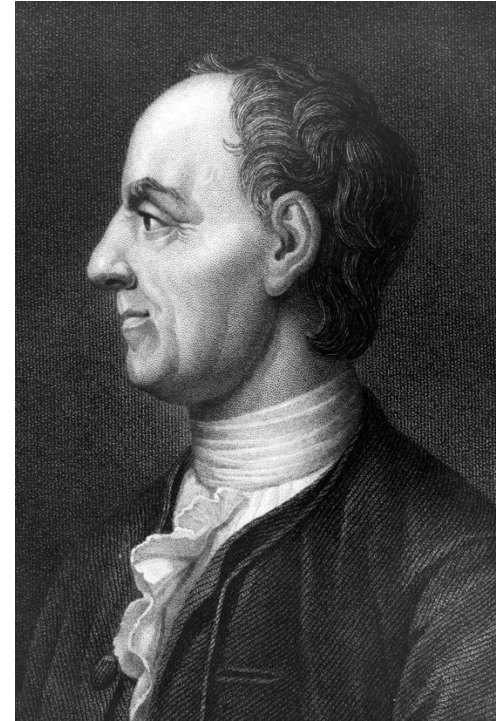
$$x(t) = Ce^{at}$$

$$x(n) = C\alpha^n$$



Leonhard Euler

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$



Leonhard Euler
Swiss mathematician
(1707-1783)

Test for:

- Periodicity
- Even Odd
- Energy, Power

Signals: Elementary

Unit ramp $r(t)$

- Test for:
- Periodicity
 - Even Odd
 - Energy, Power

Signals: Elementary

Gate function $g(t)$

Test for:

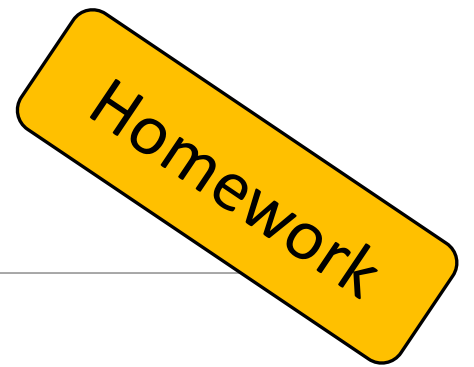
- Periodicity
- Even Odd
- Energy, Power

Signals: Elementary

Signum function $\text{sgn}(t)$

Exercises: Do at home

Chapter 1: *Alan V. Oppenheim*
Basic Problems





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