Queen Mary School Hainan
Queen Mary University of London

QHP5701 Exploratory Data Analysis

Signals

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#Ref: Chapter 1, Oppenheim

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- Overview of Signals & Systems
- What are Signals?

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Overview of Signals & Systems

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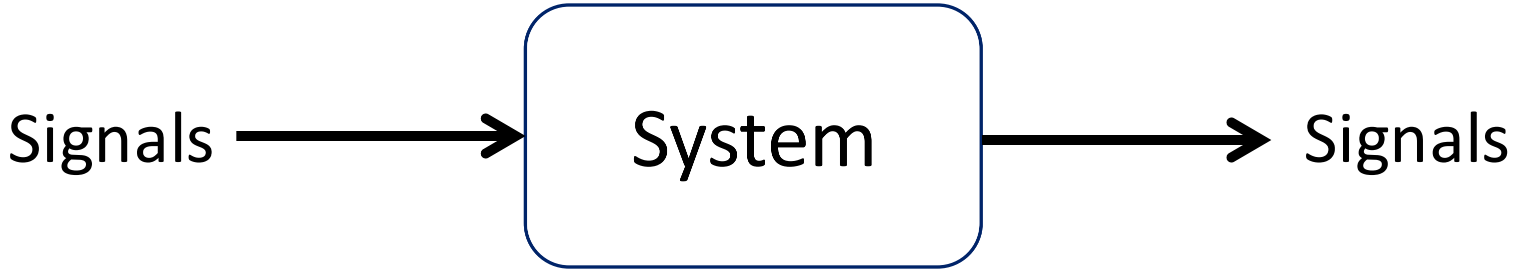
Overview of Signals & Systems

- Signals and Systems?

  Any activity of human, can be thought of interaction or an interplay of the signals & systems

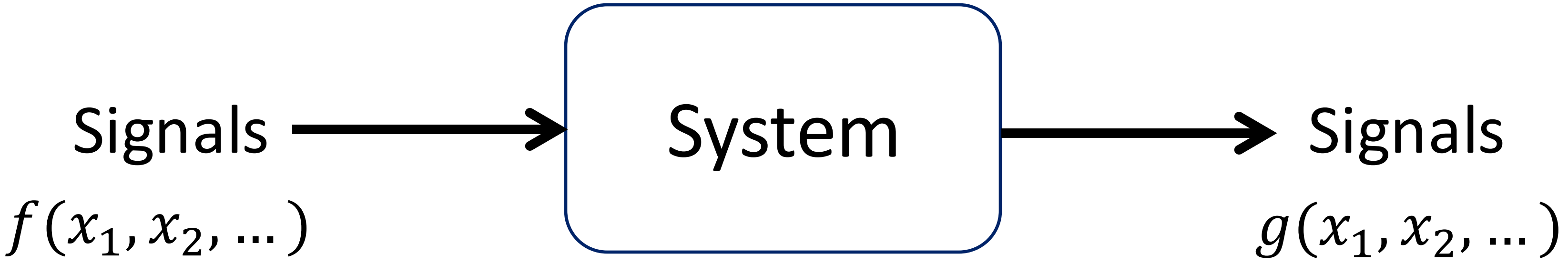
- A system as a block:
  - **is a meaningful** interconnection of physical devices or components
  - is an interconnection of subsystems, which are composed of physical devices or components
- A system by itself cannot achieve anything, it must be closely related to signals



Overview of Signals & Systems

- Signals and Systems?

  Any activity of human, can be thought of interaction or an interplay of the signals & systems



*A **signal**, in general, is a function of one or more independent variables*

System takes a signal, e.g. x(t), as input and produced more desirable output y(t)

e.g. Signal processing blocks

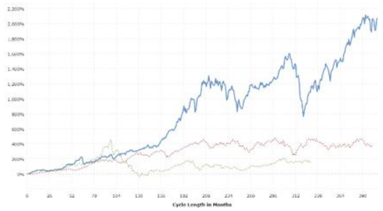
Signals

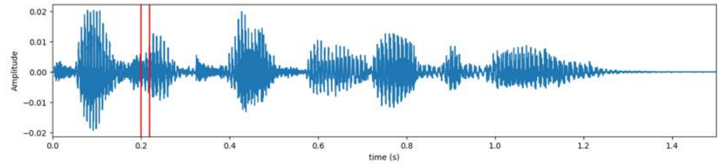
*A **signal**, in general, is a function of one or more independent variables*

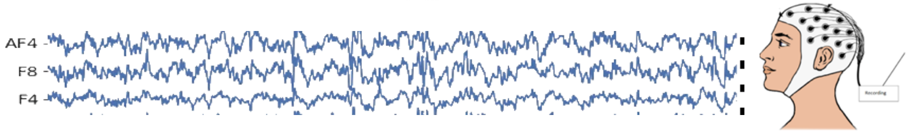
$$f(x_1, x_2, \dots), \quad x(t)$$

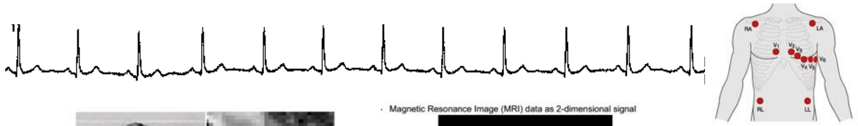
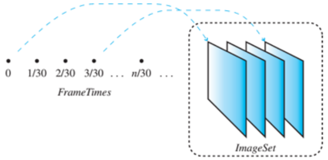
Examples

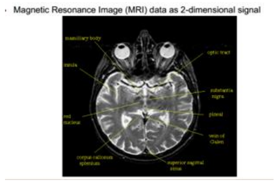
- Temperature in Hainan: x(t)
- Speech/audio: x(t)
- EEG, ECG, x(t)
- Image I(x,y)
- Video V(x,y,t)





Signals

*A **signal**, in general, is a function of one or more independent variables*

$f(x_1, x_2, \ldots), \quad x(t)$

Anything that changes over one or more dimensions can be thought as a signal

Examples of Signals & Systems

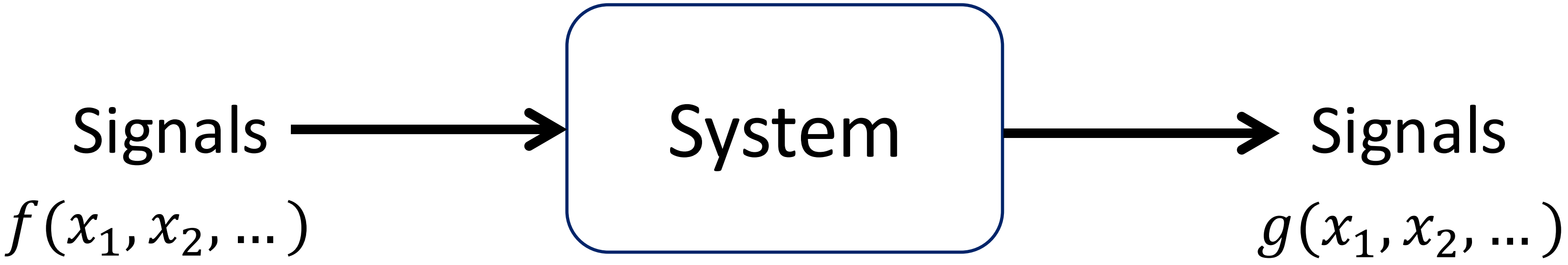
Signals $\longrightarrow$ System $\longrightarrow$ Signals

$f(x_1, x_2, \ldots)$ $\quad$ $g(x_1, x_2, \ldots)$

Examples

- Electrical Circuit
- Motor
- Heart Monitor
- …
- …

Signals & Systems: Problems



<u>Analysis Problems</u>

- Given: Input signal and system   Find: output signal

<u>Design/Synthesis Problems</u>

- Given: Input signal and desired output signal   Design: System

Signals & Systems: Why?

- Foundation of many engineering fields
- Provides <u>tools to analyse signals and systems</u>
- Used in variety of other fields
- Laydown foundations for systems design

Note: Most of slides will be empty for in-class computations

Since we are following a text-book heavily, slides, will not include all the mathematical computations and details. Please check the text-book, for details.

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Signals

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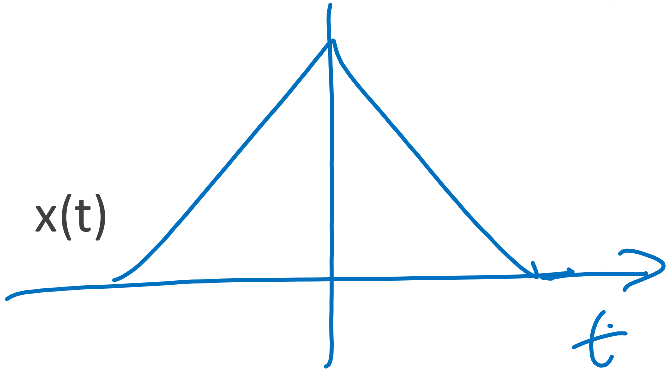
Signals:

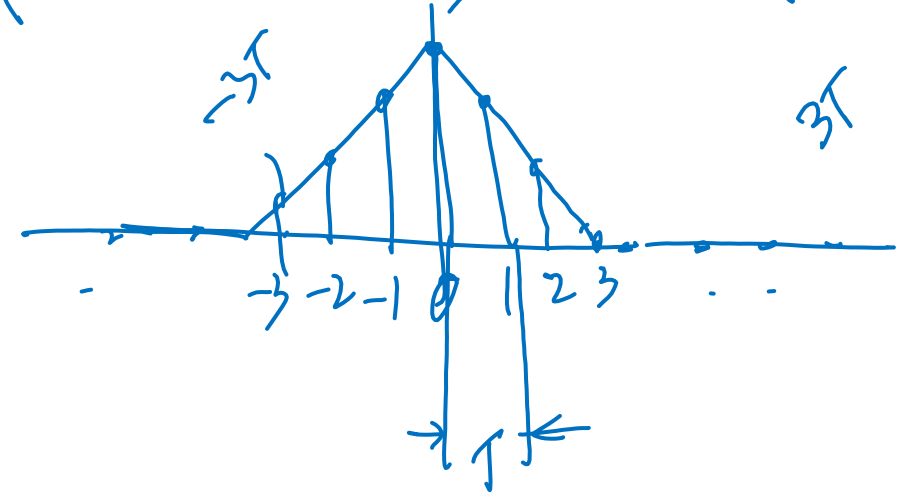
- x(t): continues-time signal
- x(n): discreet-time signal

Reconstruction of Signal

- x(t): continues-time signal
- x(n): discreet-time signal

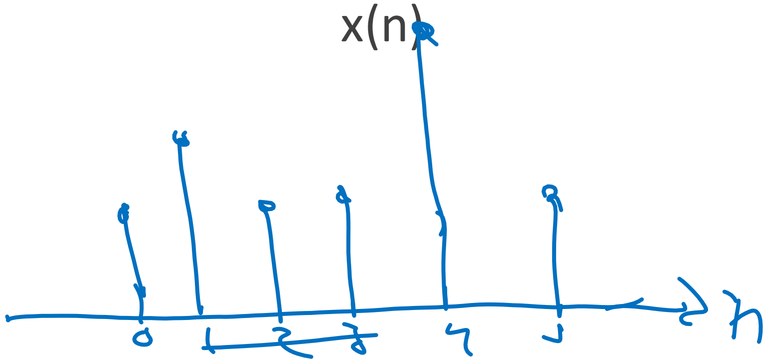
$x(t) \quad t \in \mathbb{R}$ $\qquad$ $x(n) \quad n \in \mathbb{Z}$





T

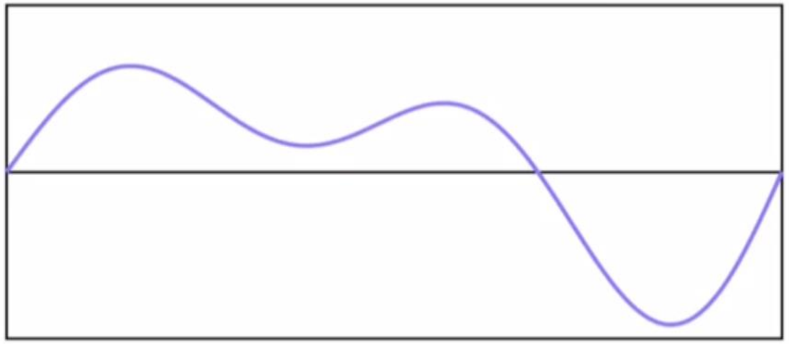
$x(n) = [1 \ 2 \ 1 \ 1 \ 3 \ 1]$



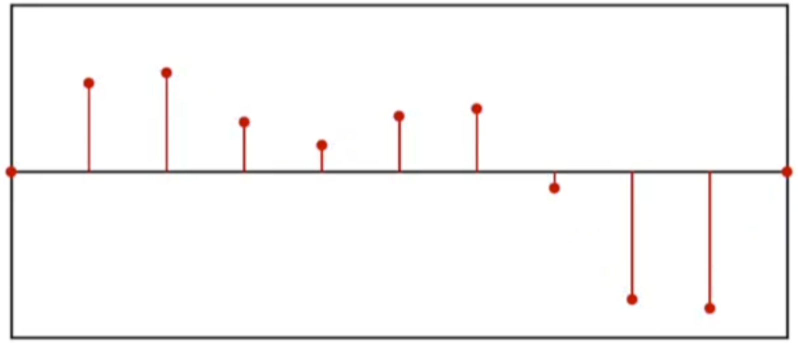
Reconstruction of Signal

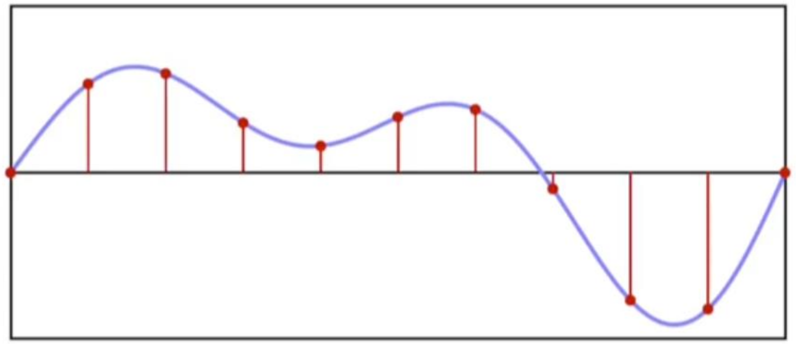
- x(t): continues-time signal
- x(n): discreet-time signal

x(t)



x(n)





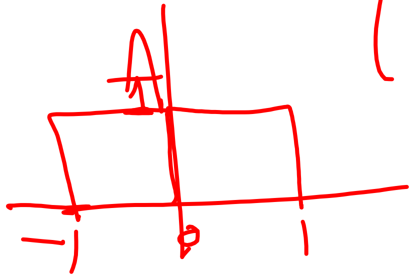
Signals: Transformation

- **Folding**, time-reversal: x(-t), x(-n)
- **Shifting**, delayed, advanced: $x(t \pm T)$, $x(n \pm N)$
- **Scaling**, compression and expansion: $x(\alpha t)$, $\boxed{x(kn)}$
- **Amplitude Scaling**, amplification and attenuation: Ax(t)

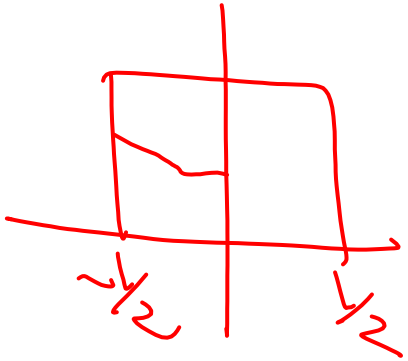
$-1 < 2t < 1$

$-\frac{1}{2} < t < \frac{1}{2}$

$A\,x(t) = \begin{cases} 1 > t > -1 = 1 \\ 0 \quad \text{else} \end{cases}$



$x(2t)$

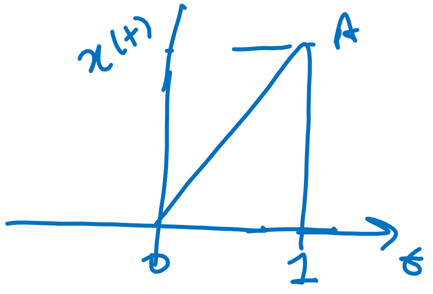


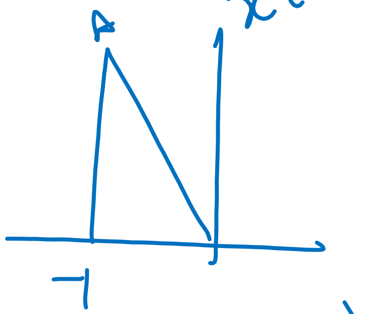
$\begin{cases} 1 \quad -1 < 2t < 1 \\ 0 \quad\quad 0 \text{ else} \end{cases}$

e

Computation

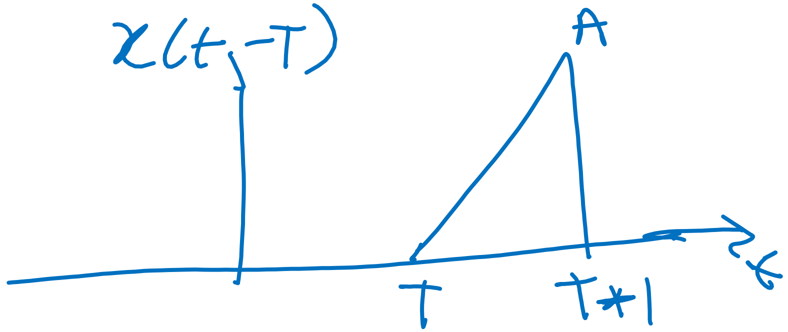
$x(t) \longrightarrow x(-t)$





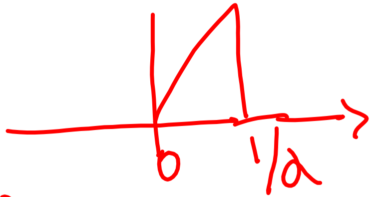
$x(t) \longrightarrow x(t-T)$ Delayed

TER



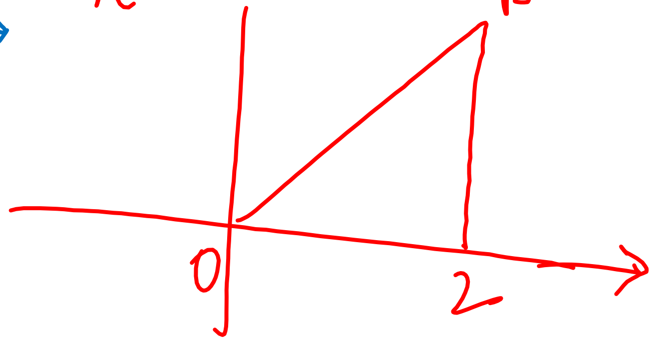
$x(t+T)$ Advanced

$x(t) \longrightarrow x(at)$

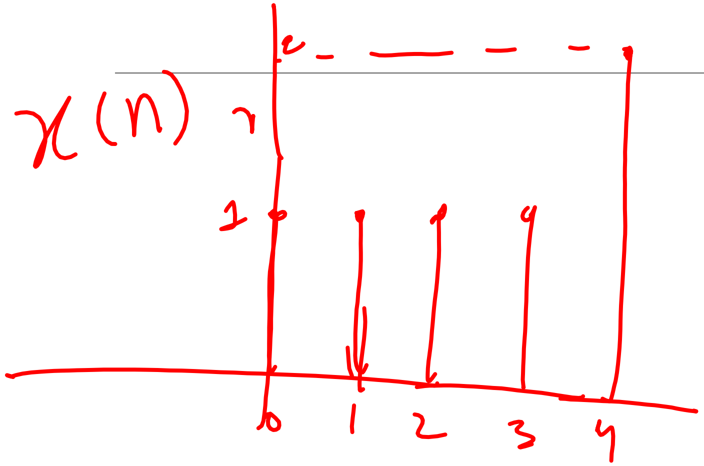


$x(at)$ —

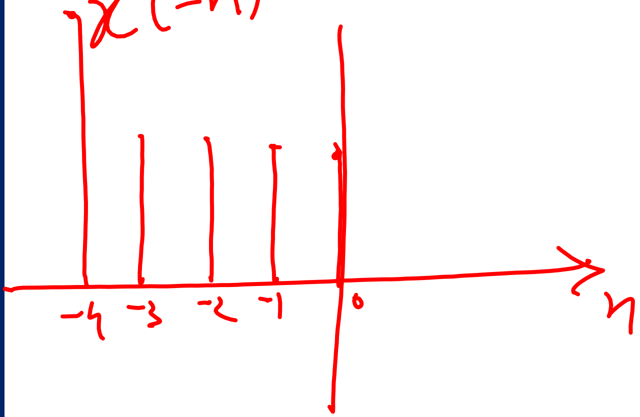
$x(t/2)$



$x(n)$



$x(-n)$



$x(n) \to x(n \pm N)$

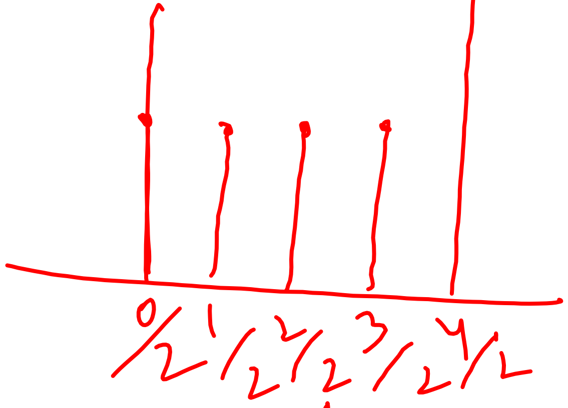
$x(n) \to x(n + 1/2)$ ✗

$x(n+3)$

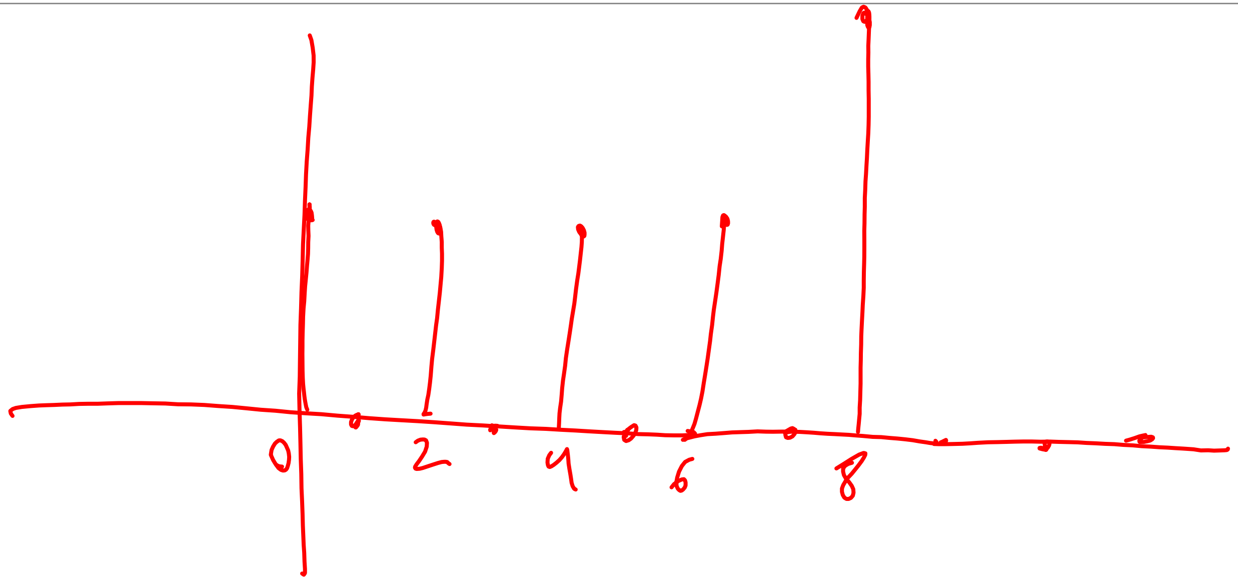
$$x(n) = \begin{cases} 1 & n=0 \\ 1 & n=1 \\ & =2 \\ & =3 \\ 2 & n=4 \end{cases}$$

$x(2n)$ — decimation

downsampling



$x(n)$

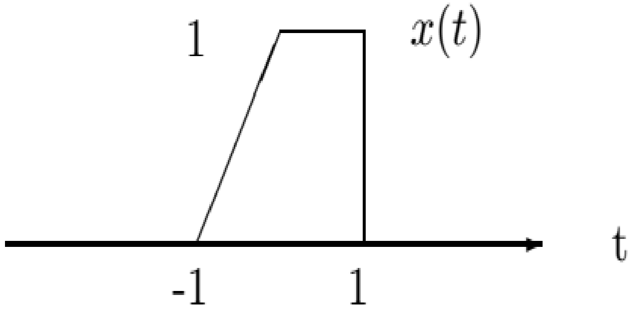


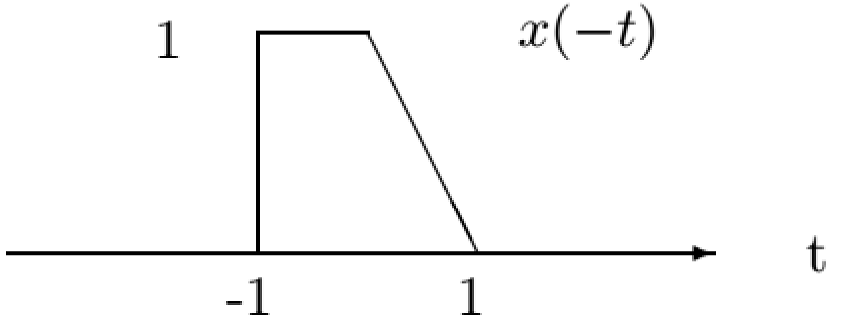
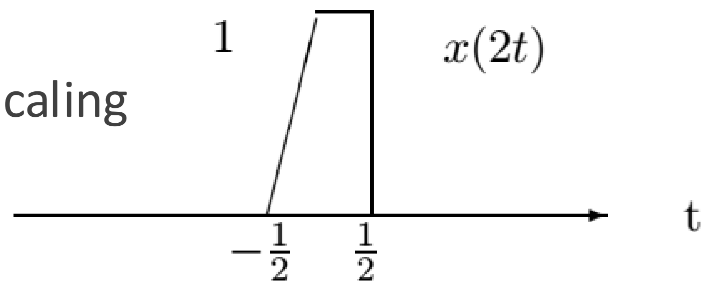
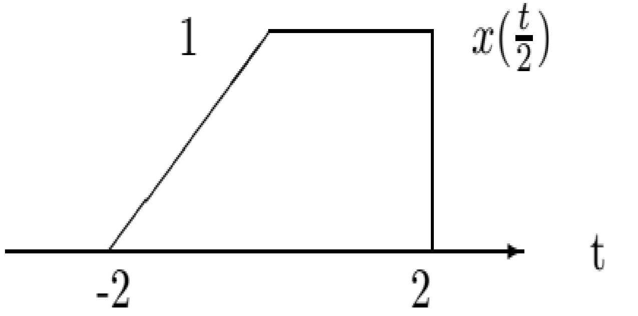
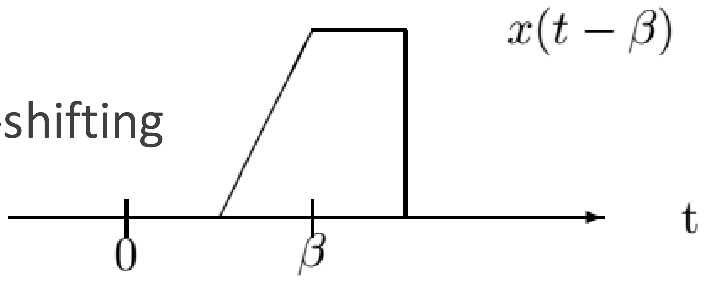
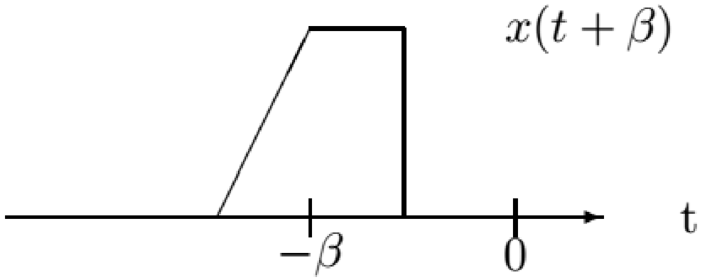
$x(n/2)$

← up sampling

—

Examples



time-reversal		
time-scaling		
time-shifting		

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Signals: Classification of signals

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#Ref: Chapter 1, Oppenheim

Signals: Classification

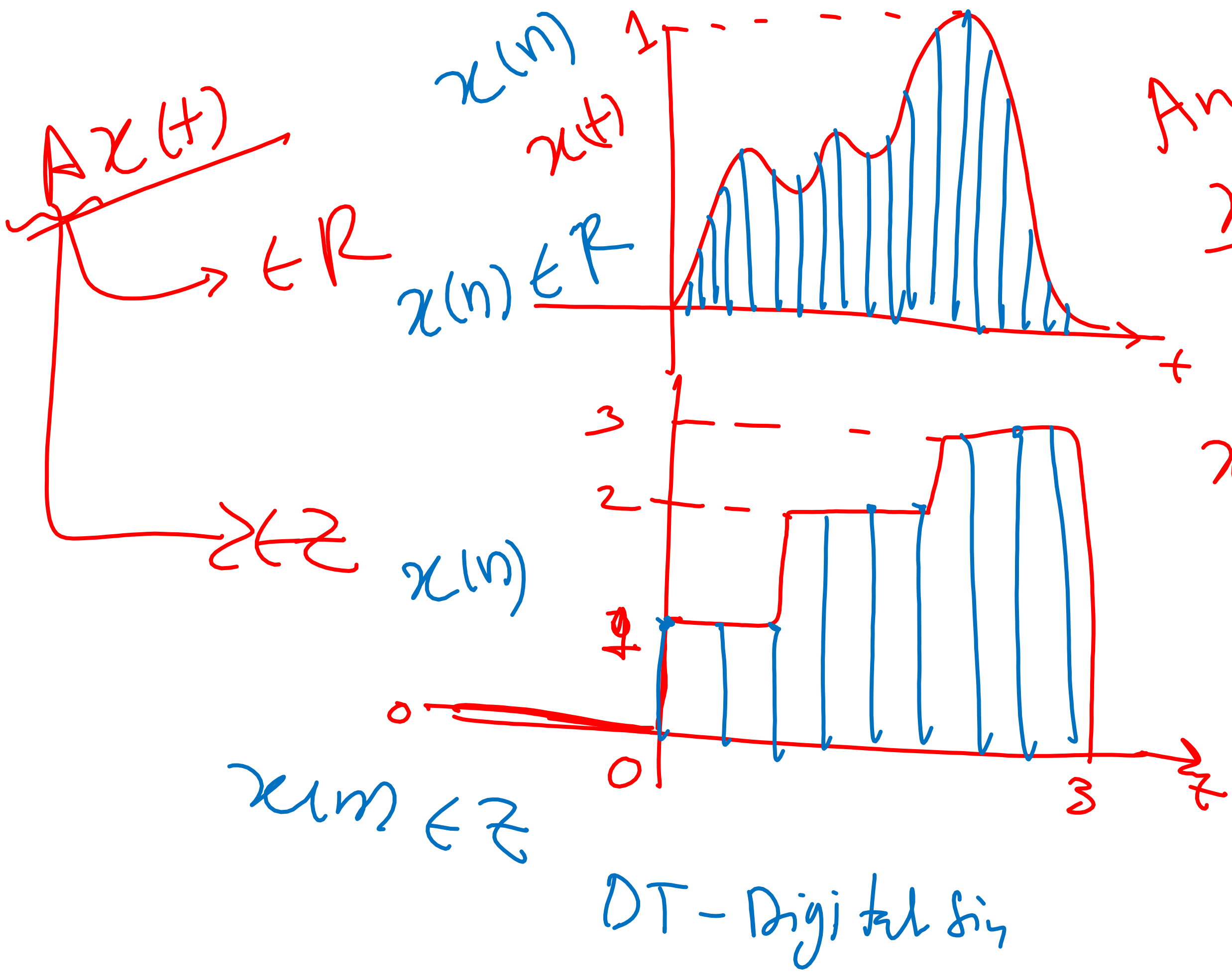
- Classification of Signals: Properties (attributes/features) of signals
  - Continues & Discrete
  - Analog & Digital
  - Periodic & Aperiodic
  - Even & Odd
  - Energy & Power
  - Real & Complex
  - Deterministic & Random & Chaotic signal

Continues & Discrete time signals

CT $x(t)$ $t \in \mathbb{R}$

DT $x(n)$ $n \in \mathbb{Z}$

Analog & Digital signals

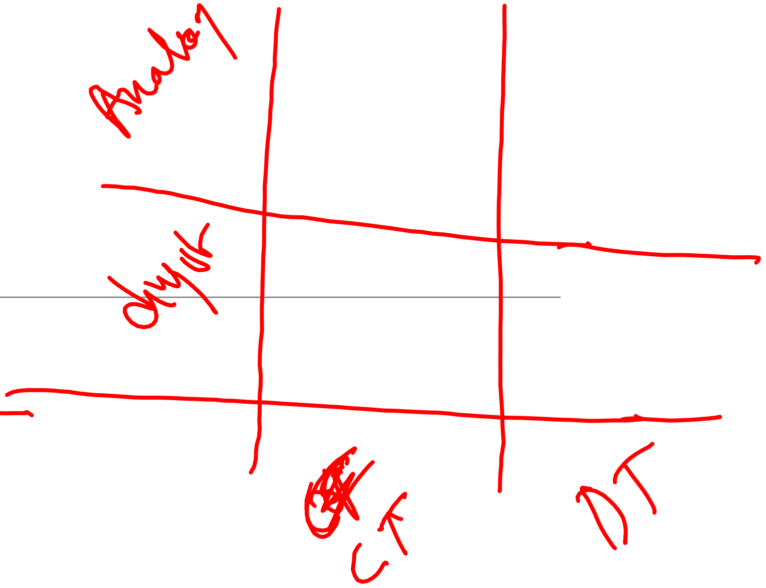


Analog

$x(t) \in R$

$x(t) \in Z$

$x(t) \in [0, 1, 2, 3]$



Cont. & Discrete | Analog & Digital

Which one is which?

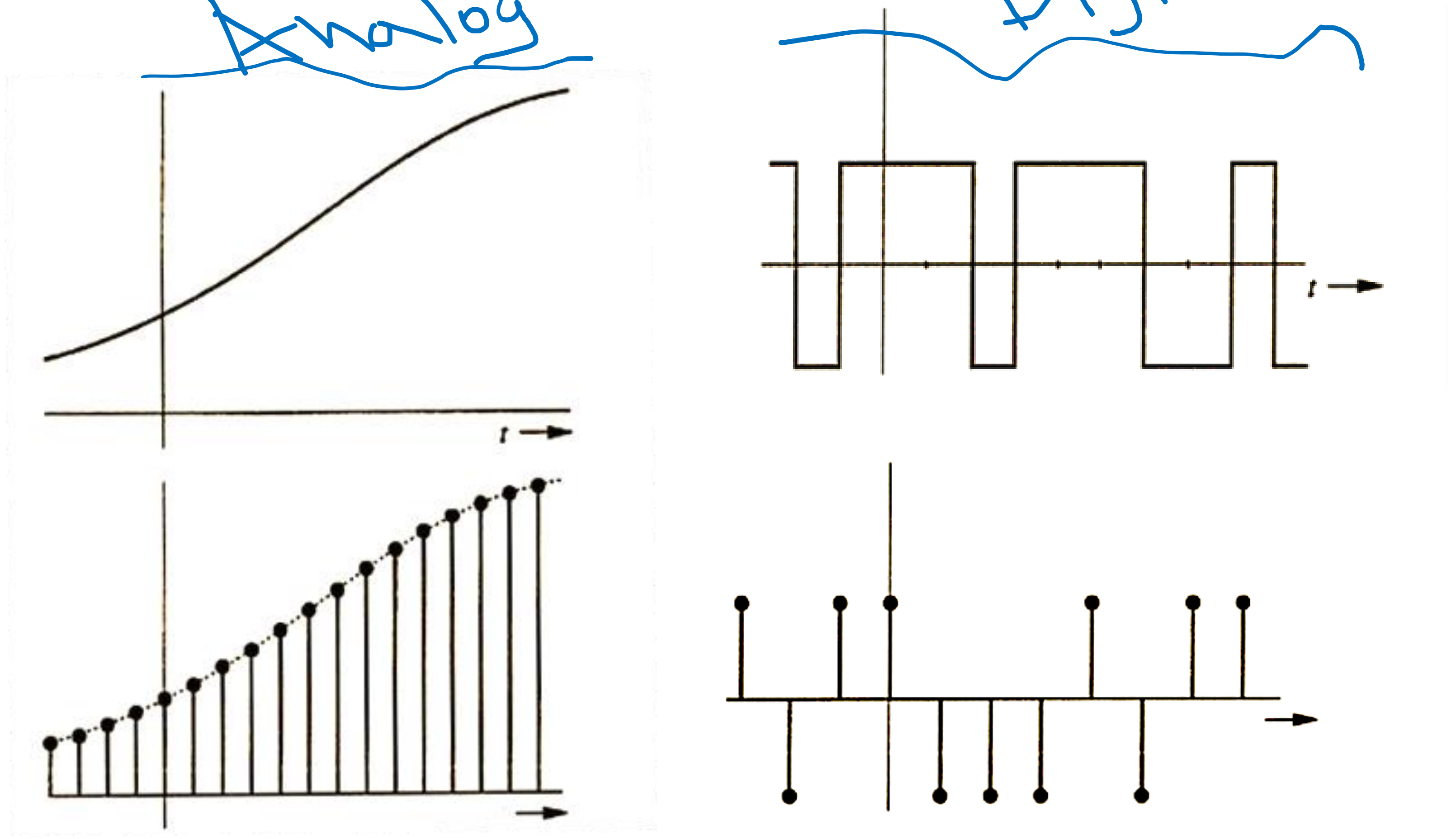
- Digital, CT
- Analog, CT
- Digital, DT
- Analog, DT

Analog

Digital

CT

DT

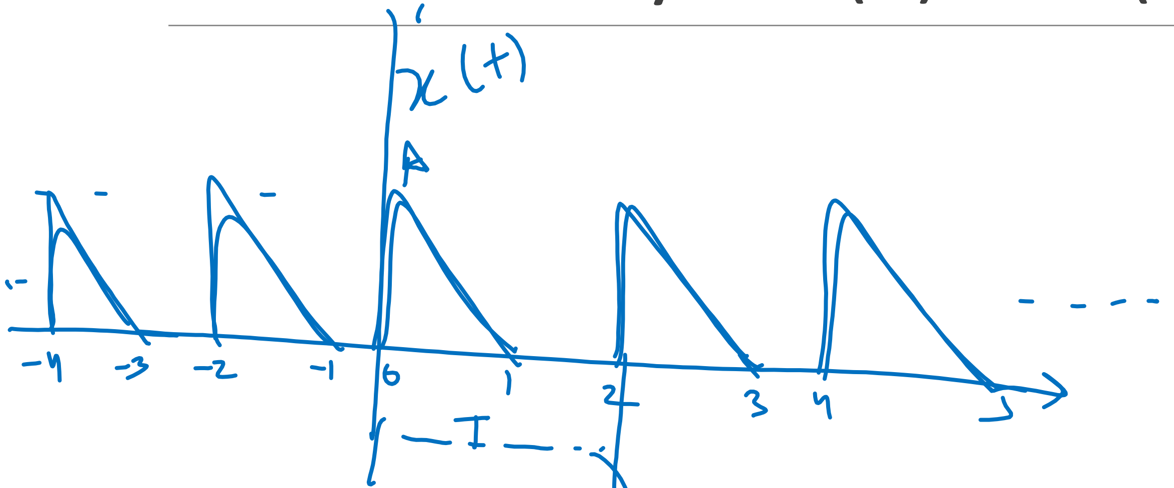


Periodic & Aperiodic

Periodicity: $x(t) = x(t + T)$

$$x(t) = x(t+2) = x(t+4) = x(t+6) \quad \vdots \quad x(t+mT)$$

$$x(t) = x(t+2) \quad T = 2$$



$$x(t+2) \qquad T = 2$$

fundamental period $= \min\{mT\}$

$= \min\{\varphi\}$

Periodic & Aperiodic

Periodicity: x(t) = x(t + T)

$\omega = 2\pi f$

$x(t) = \cos(\omega t + \phi)$

$$x(t) = x(t + mT)$$

$$x(t) = x(t + \alpha)$$

T_o

$$fundamental\ period = T_o = \min\{\alpha\}$$ sec

$$fundamental\ frequency = f = \frac{1}{T_o}\ cycles/s\ \text{(Hz)}$$

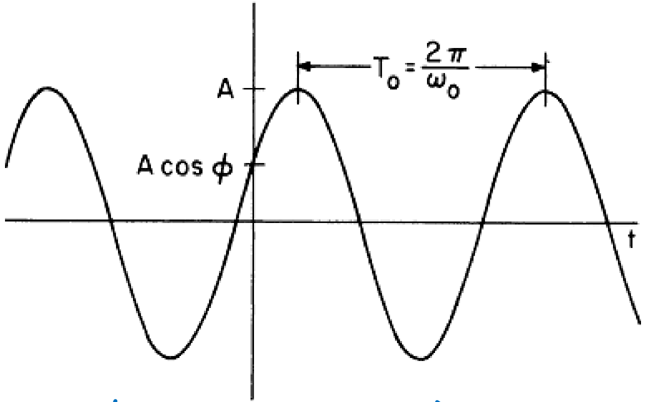
integer

$$x(n) = x(n + N)$$

Periodic & Aperiodic

Periodicity: $x(t) = x(t + T)$

$\underline{x(t) = A\cos(\omega_0 t + \phi)}$



$x(t) = \sin\left(\frac{4\pi}{3}t + 2\pi t\right)$

$T_0 = \frac{2\pi}{\omega_0} = \frac{\cancel{2\pi}}{\frac{10\cancel{\pi}}{3}} = 3/5$ sec

$$\boxed{T_0 = \frac{2\pi}{\omega_0}}$$

$A\cos(\omega_0 t + \phi)$

$\omega_0 = 2\pi f_0$

$f_0 = \frac{\cancel{2\pi}}{\cancel{\omega_0}} \quad \frac{\omega_0}{2\pi}$

$T_0 = \frac{1}{f_0} = \frac{2\pi}{\omega_0}$

$x(t) = \cos(2\pi t)$

$\omega_0 = 2\pi$

$x(t) = \sin\left(\frac{3}{4}\pi t\right)$

$T_0 = \frac{2\pi}{\frac{3}{4}\pi}$

$= \frac{2\times 4}{3}$

$\boxed{T_0 = \frac{8}{3}\text{ s}}$

Periodic & Aperiodic

Periodicity: x(t) = x(t + T)

$x(t) = \cos(\omega_o t)$ $T_o = 2\pi/\omega_o$

$x(t) = \cos(2\pi t)$ $T_o = 1\,s$

$x(t) = \cos(t/6)$ $T_o = 12\pi\,s$

$x(t) = \cos(2\pi t) + \cos(8\pi t)$

Periodicity: $x(t) = x(t + T)$

Periodic & Aperiodic

Periodicity: x(n) = x(n + N)

$x(n) = \cos(\Omega_o n)$

$x(n) = \cos(2\pi n)$

$\cancel{x(n)} = \cos(n/6)$

Not periodic

$$\boxed{N_0 = \frac{2\pi}{\Omega_0} \times m}$$

$N_0 = \frac{2\pi}{2\pi} m = 1$ sample $m=1$

$N_0 = \frac{2\pi}{1/6} m = 12\pi m$

$x(n) = \cos(2$

$$x(n) = \cos\left(\frac{5\pi}{31} n\right)$$

$$N_0 = \frac{2\pi}{\Omega_0} m = \frac{2\pi \times 31}{5\pi} m = \boxed{\frac{62}{5} m = N_0}$$

$\frac{N_0}{m}$ = rational number

$\frac{N_0}{m} = \frac{62}{5}$

$m = 5$

$\boxed{N_0 = 62}$

Periodicity: $x(n) = x(n + N)$

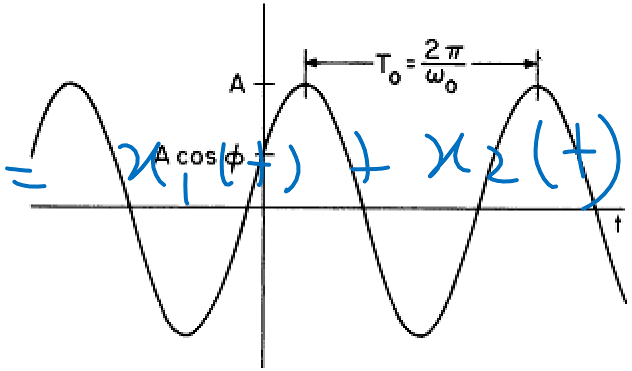
Periodicity: Examples

$$\boxed{T_o = \frac{2\pi m}{\omega_o} \Rightarrow \text{period} = \frac{2\pi}{\omega_o}}$$

$x_1(t) \rightarrow T_1$

$$x(t) = A\cos(\omega_o t + \phi)$$

$x_2(t) \rightarrow T_2$



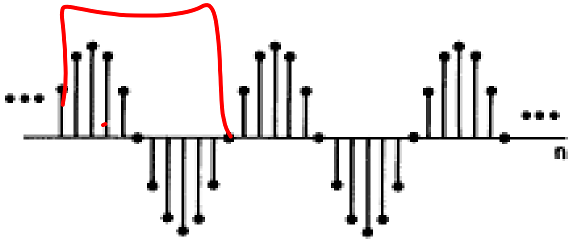
$y(t) = x_1(t) + x_2(t)$

$$\boxed{\Omega_o = \frac{2\pi m}{N}}$$

$$x[n] = A\cos(\Omega_o n + \phi)$$

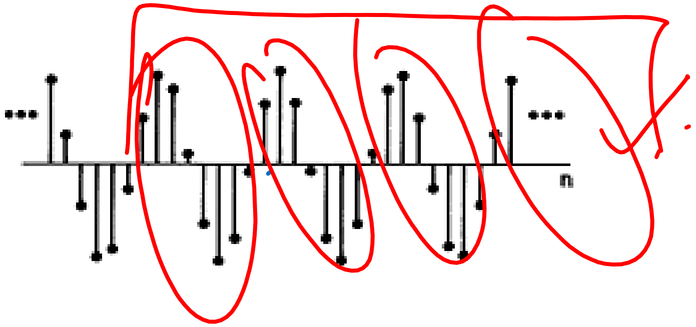
$\Omega_o = \frac{2\pi}{12}$

$\phi = 0$



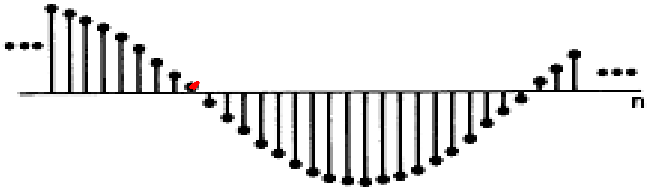
$\Omega_o = \frac{8\pi}{31}$

$\phi = 0$



$\boxed{\Omega_o = \frac{1}{6}}$

$\phi = 0$



$\Omega_o = \frac{2\pi}{12}$

$N_o = \frac{2\pi \times 12}{2\pi}$

$\Omega_o = \frac{8\pi}{31}$

$N_o = \frac{2\pi \cdot 31\, m}{8\pi}$

$N_o = \frac{31}{4} m$

$N_o = 31$

$m = 4$

Homework

Periodicity: Deductions!!

Deduce following with possible answers as:

Always Periodic, Not Always Periodic, Always Aperiodic, Not Always Aperiodic

Answers will be different for Continues Time and Discrete Time Signals So answer separately for Continues and Discrete

Cases		Continues	Discrete	
Periodic + Periodic	=	________________	________________	(+ and -)
Periodic + Aperiodic	=	________________	________________	
Aperiodic + Aperiodic	=	________________	________________	
Periodic x Periodic	=	________________	________________	
Periodic x Aperiodic	=	________________	________________	
Aperiodic x Aperiodic	=	________________	________________	

Heinrich Hertz

Measure for frequency

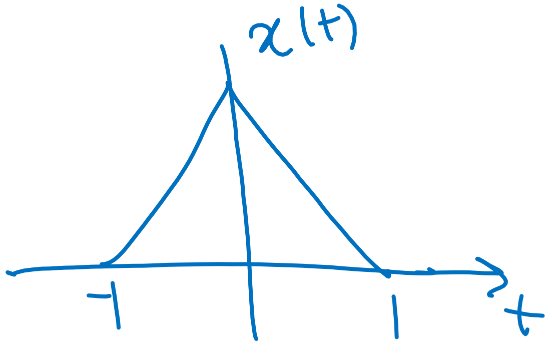
$cycles/s$ = Hertz = Hz

- German physicist, most of work on radio waves and electromagnetism
- Died very young, at age of 37

Heinrich Rudolf Hertz
German physicist
(1857-1894)

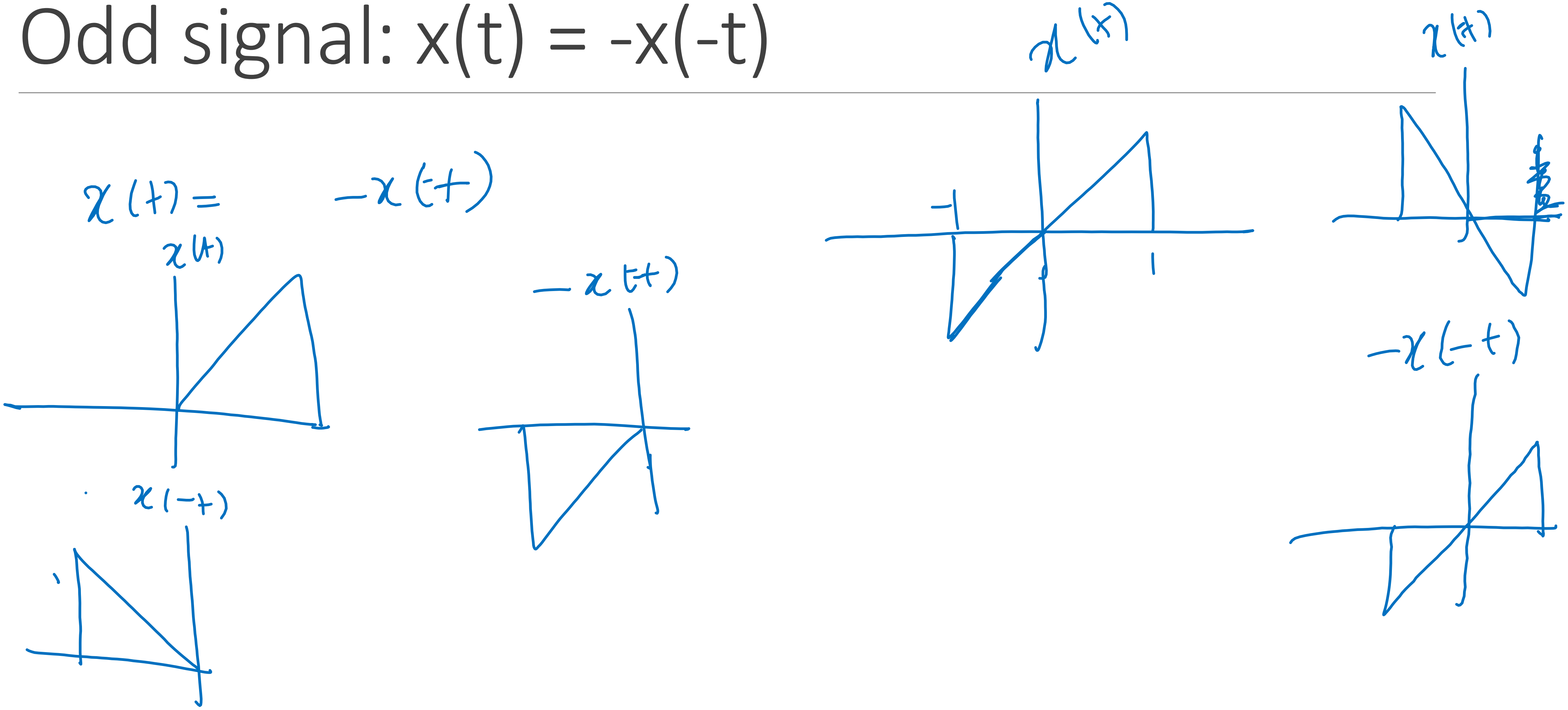
Even signal: $x(t) = x(-t)$

$x(t) = x(-t)$



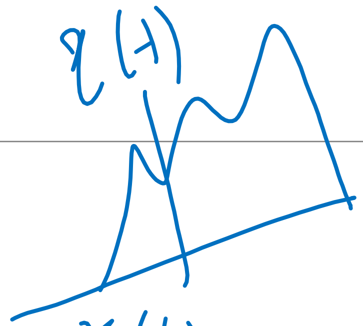
Even & Odd

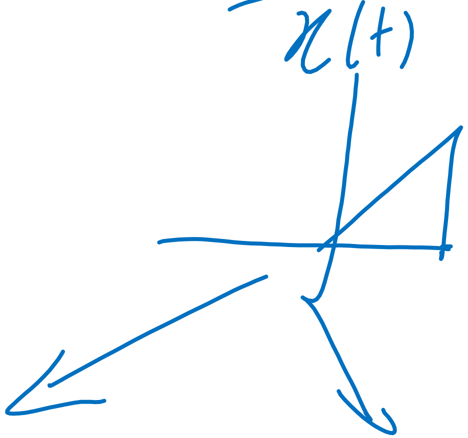
Odd signal: x(t) = -x(-t)



Even & Odd

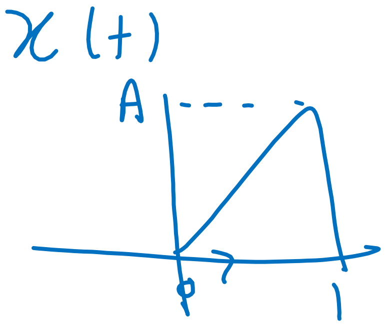
$$x(t) = Ev\{x(t)\} + Od\{x(t)\}$$



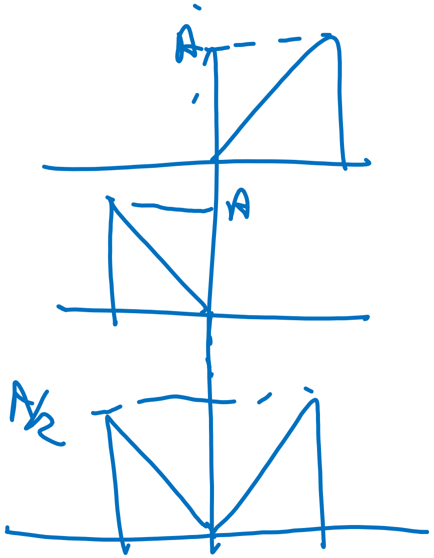


$$Ev\{x(t)\} = \frac{x(t) + x(-t)}{2}$$

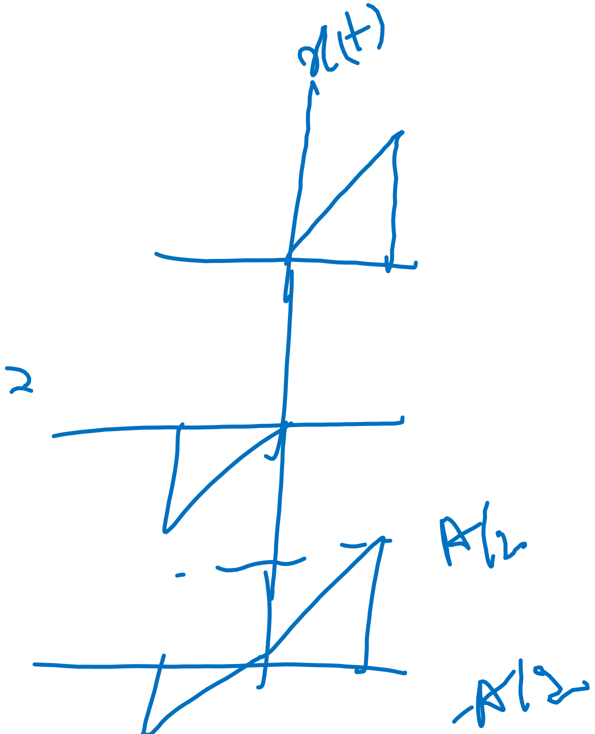
$$Od\{x(t)\} = \frac{x(t) - x(-t)}{2}$$



$Ev\{x(t)\} =$



$Od\{x(t)\} =$



Energy & Power

$v = iR$

$i = v/R$

- Instantaneous power p(t)
- Total energy over duration $\mathrm{E} = \int_{t_1}^{t_2} |x(t)|^2 dt$
- Average power over duration
- Total energy $\mathrm{E}_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$
- Total power

$$\mathrm{E} = \sum_{n=n_1}^{n_2} |x(n)|^2$$

$p(t) = i(t)\, v(t)$

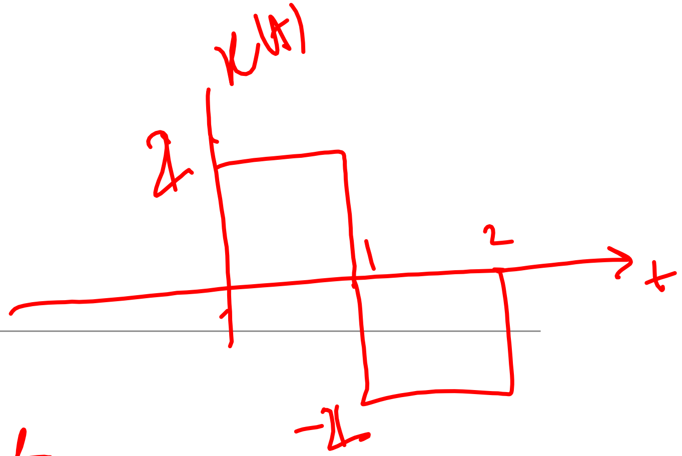
$$\boxed{p(t) = \frac{1}{R} v^2(t)}$$

$$E = \int_{t_1}^{t_2} p(t)\, dt$$

$$P = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} v^2(t)\, dt$$

$$E = \int_{t_1}^{t} v^2(t)\, dt$$

Energy & Power



Total Energy

$$E_\infty = \lim_{T\to\infty}\int_{-T}^{T} x^2(t)\,dt = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$E_{01} = \int_0^1 1^2\,dt = 1$$

$$P_{01} = \frac{1}{1}E_{01} = 1$$

$$P_{02} =$$

$$E_\infty = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_0^1 1^2\,dt + \int_1^2 (-1)^2\,dt = 4+4 = 8$$

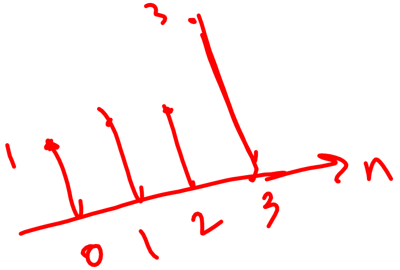
$$P_\infty =$$

Energy & Power

$$P = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} x^2(t)\,dt$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x^2(t)\,dt$$

$x(n) =$



$$E_\infty = \sum_{n=-\infty}^{\infty} x^2(n) \qquad E = \sum_{n=n_1}^{n_2} x^2(n)$$

$$= 1^2 + 1^2 + 1^2 + 3^2 = 3 + 9$$

$$= 12$$

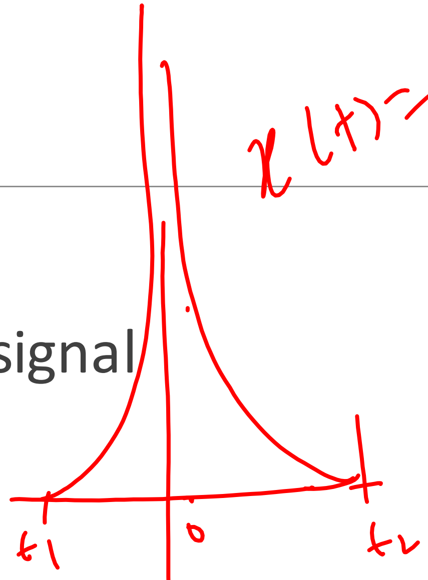
$$P_\infty = \frac{12}{4} = 3$$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{N} x^2(n)$$

Energy & Power Signal

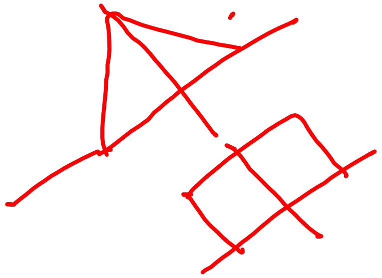
Energy Signal

- A signal with finite energy; $\boxed{E < \infty}$, is called energy signal
- it has 0 average power, P=0



$x(t) = \sin(2\pi t)$

$E_\infty = \infty$



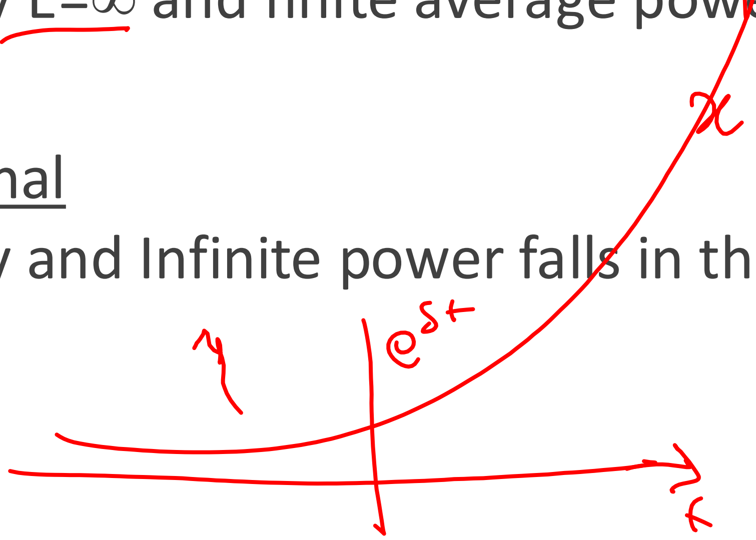
Power Signal

- A signal with infinite energy E=∞ and finite average power P<∞ are called power signal

$x(t) = 1/|t|$

Neither Energy nor Power signal

- A signal with infinite energy and Infinite power falls in this category
- $\boxed{E = \infty \ \& \ P = \infty}$



Real & Complex signal

$$x(t) = \cos(2\pi t) + j\sin(2\pi t)$$

Deterministic & Random & Chaotic signal

Chaos Theory

$x(t) = 2\sin(3\pi t) + \cos(4\pi t) + \cdot$

$t = 10000$

R

QHP5701 Exploratory Data Analysis

Signals: Elementary signals

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#Ref: Chapter 1, Oppenheim

Signals: Elementary

- Unit Step
- Unit Impulse function, Direc Delta
- Generalised Exponential
  - Exponential
  - Sinusoidal
  - Complex Exponential
- Unit Ramp
- Gate function – Square
- Signum function

Test for:

- Periodicity
- Even Odd
- Energy, Power

Signals: Elementary

Test for:
- Periodicity
- Even Odd
- Energy, Power

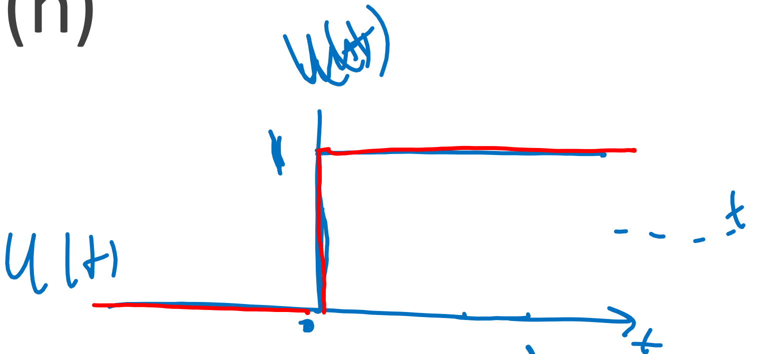
$x(t) = \cos(2\pi t)\, u(t)$

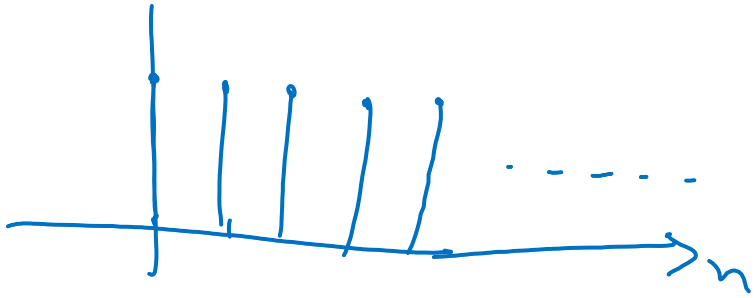
$u(t) = Ev\{u(t)\} + Od\{u(t)\}$

$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & else \end{cases}$

Unit step u(t), u(n)

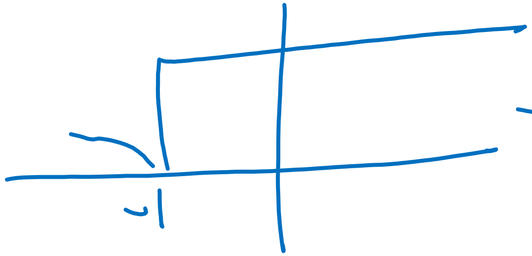
$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$

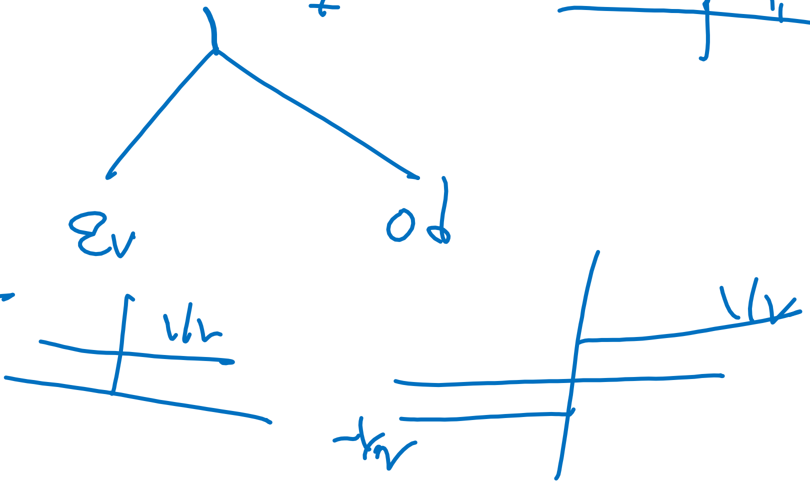




$u(t)\big|_{t=0} =$

$u(t+1)$





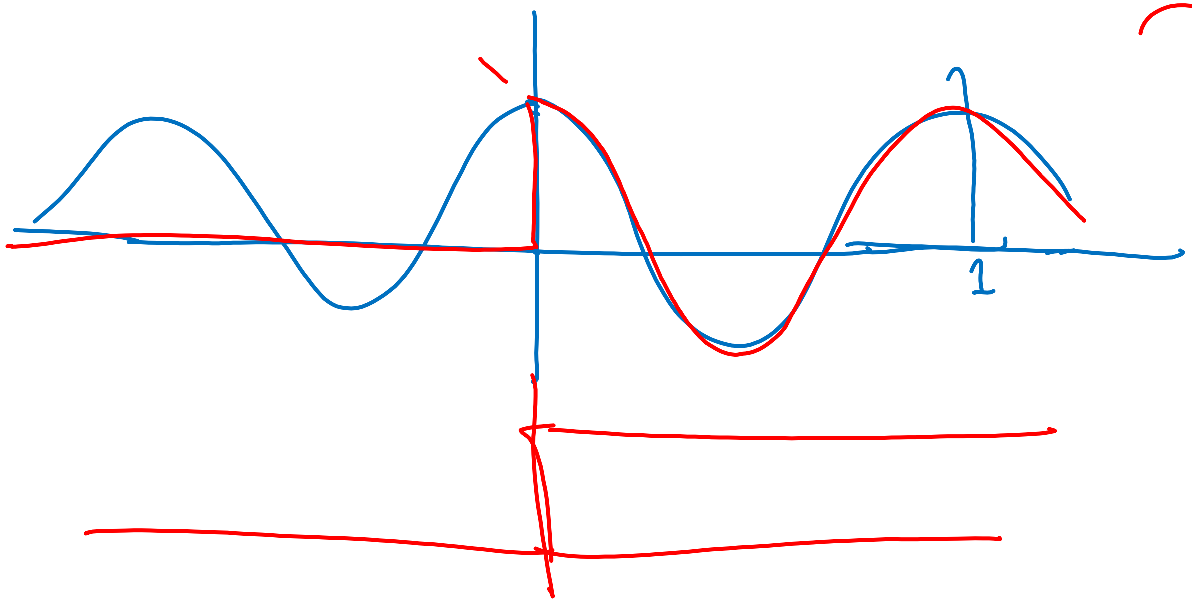
Test for:
- Periodicity
- Even Odd
- Energy, Power

Signals: Elementary

Unit step u(t), u(n)

$x(t) = \cos(2\pi t)\, u(t)$

$\cos(2\pi t) + u(t)$



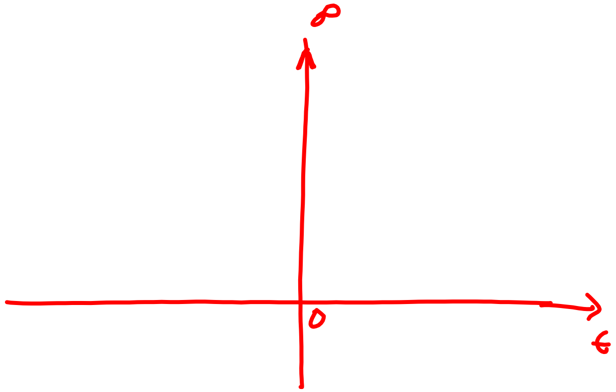
Test for:

- Periodicity
- Even Odd
- Energy, Power

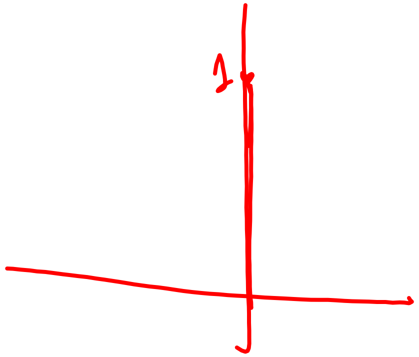
Signals: Elementary

Unit Impulse function $\delta(t), \delta(n)$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

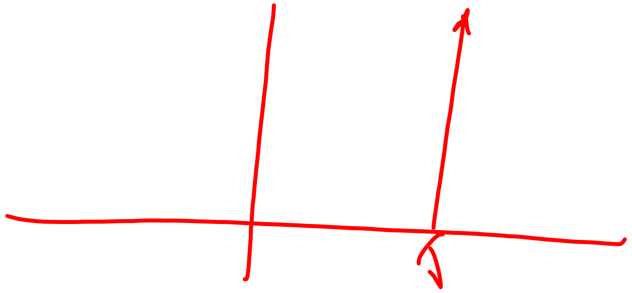


$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



$$\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & t \neq 0 \end{cases}$$

$$\delta(t - T)$$



$$\sum_{-\infty}^{\infty} \delta(n) = 1$$

Test for:
- Periodicity
- Even Odd
- Energy, Power

Signals: Elementary

- Practicality of u(t) and $\delta(t)$
- Relation of u(t) and $\delta(t)$ also in DT

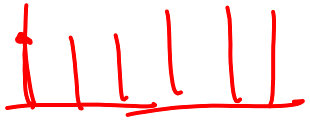
$$\mathrm{u(t)} = \int_{-\infty}^{t} \delta(\tau)d\tau$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

$$\delta(\mathrm{t}) = \frac{du(t)}{\mathrm{dt}}$$

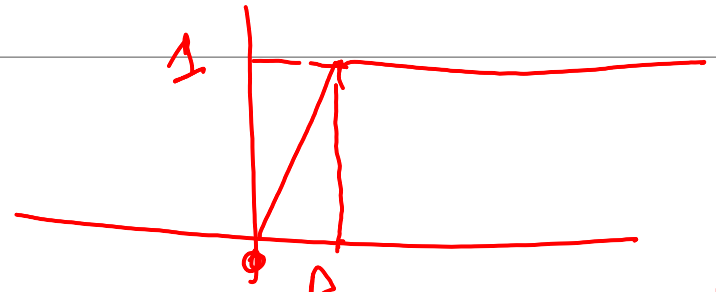
$$\delta(t) = \frac{d\,u(t)}{dt}$$

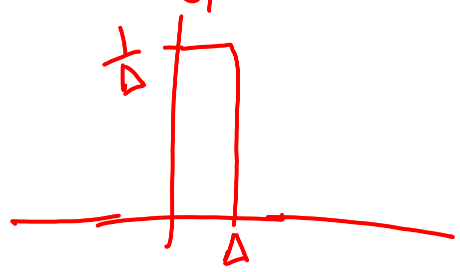
$u(n) =$



$$\delta(n) = u(n) - u(n-1)$$

$$u(n) = \sum_{m=-\infty}^{n} \delta(n-m)$$





$$\int_{0^-}^{0^+} \delta_1(t)\,dt = 1$$

$$u(t) = \lim_{\Delta \to 0} u_1(t)$$

$$\delta(t) = \lim_{\Delta \to 0} \delta_1(t)$$

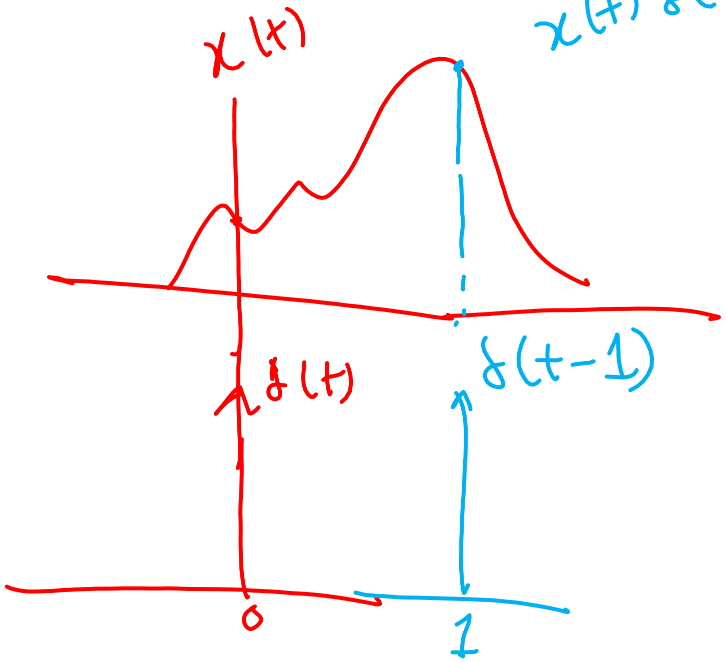
Test for:

- Periodicity
- Even Odd
- Energy, Power

Signals: Elementary

- Use of delta $x(t)\delta(t)$
- Sampling/Sifting property of delta

$x(t)\delta(t-1)$



$y(t) = x(t)\delta(t)$

$\cancel{= \delta}$

$y(t) = x(0)\delta(t)$

$\int x(0)\delta(t)\,dt = \underline{x(0)}$

$y(t) = x(t)\delta(t-1)$

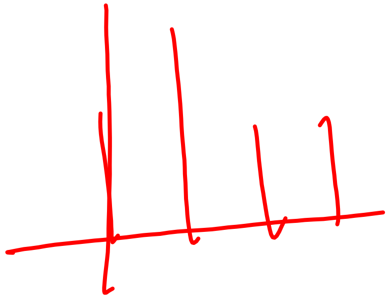
$= x(1)\delta(t-1)$

$\int x(1)\delta(t-1)\,dt = x(1)$

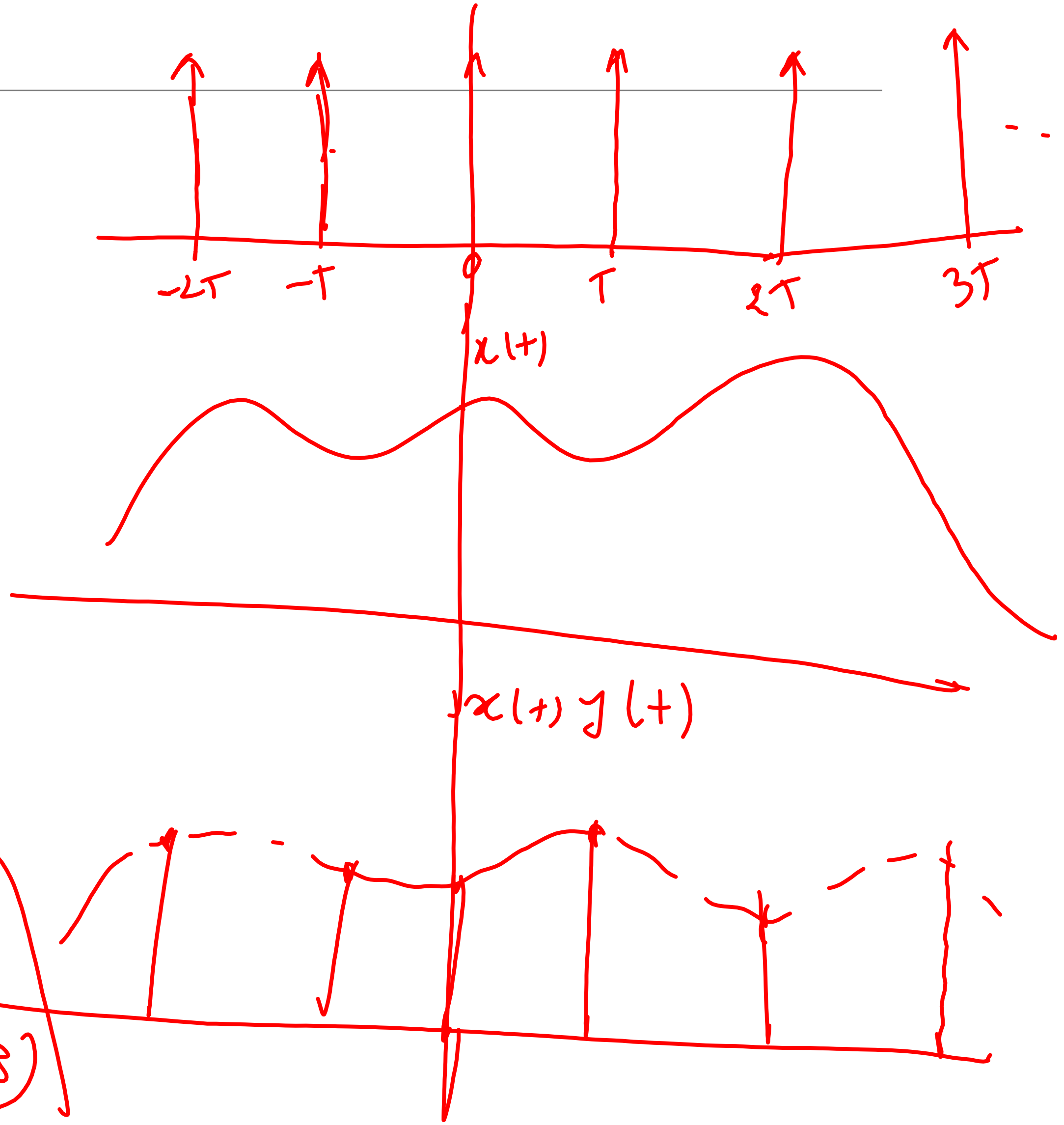
$$y(t) = \sum_{m=-\infty}^{\infty} \delta(t - mT)$$

$\delta(t)$ $\qquad \delta(n)$ $\qquad \delta(n-1)$

$$x(n) = [1 \quad 2 \quad 1 \quad 1]$$



$$\boxed{x(n) = \delta(n) + 2\delta(n-1) + \delta(n-2) + \delta(n-3)}$$



Test for:

- Periodicity
- Even Odd
- Energy, Power

Signals: Elementary

<u>Generalised Exponential function</u>

- Exponential
- Complex Exponential
- Sinusoidal

$$\boxed{x(t) = Ce^{at}}$$

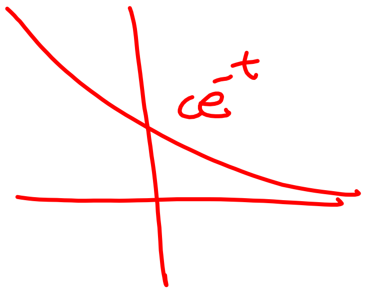
C= complex or real

a = complex or real

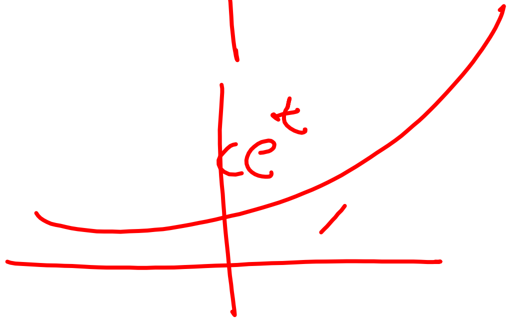
$C =$ Real $\quad a$ – Real

$a < 0$

$x(t) = Ce^{at}$



$a > 0$

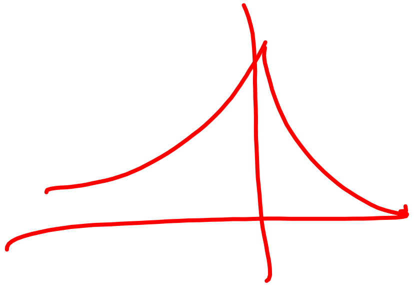


$x(t) = 5e^{-2t}$

$x(t) = \frac{1}{2}e^{+\frac{1}{3}t}$

$x(t) = e^{-2t}u(t)$

$\cancel{+ e^{+2t}u(-t)}$



Signals: Elementary

Generalised Exponential function $x(t) = Ce^{at}$

$C = \text{Real}$ $\quad a = \text{Complex}$

$a = j\omega_0$

$T_0 = \frac{2\pi}{\omega_0}$

$x(t) = Ce^{j\omega_0 t} = C\left[\cos(\omega_0 t) + j\sin(\omega_0 t)\right]$

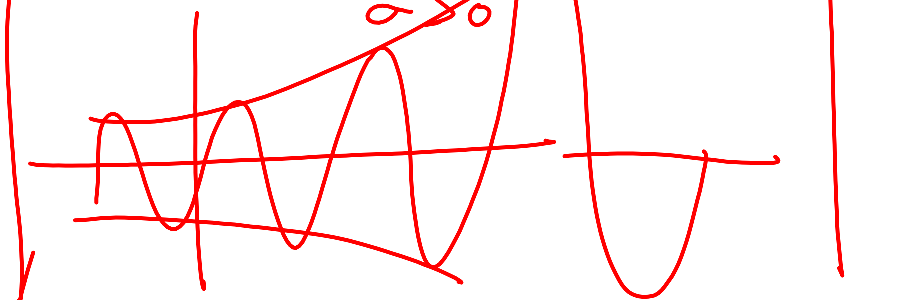
$x(t) = 5e^{j\omega_0 t}$

$= 5e^{j5\pi t}$

$T_0 = \frac{2\pi}{5\pi} = 2/5$

$x(t) = Ce^{\sigma t}e^{j\omega t}$

$\sigma > 0$

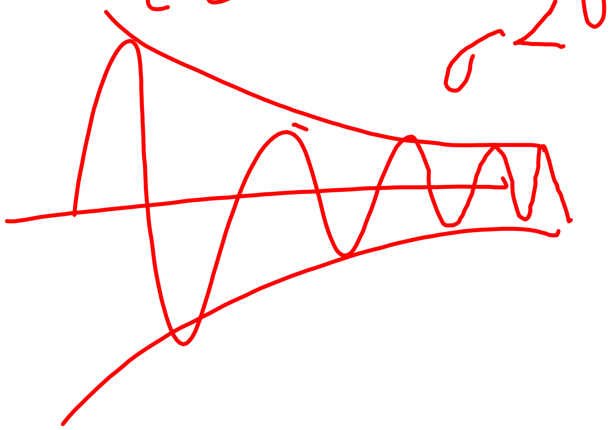


$a = \sigma + j\omega_0$

$x(t) = Ce^{(\sigma + j\omega_0)t}$

$= Ce^{\sigma t} \cdot e^{j\omega_0 t}$

$\sigma < 0$



Signals: Elementary

$C = c_1 + jc_2 \longrightarrow C = |c| e^{j\theta}$

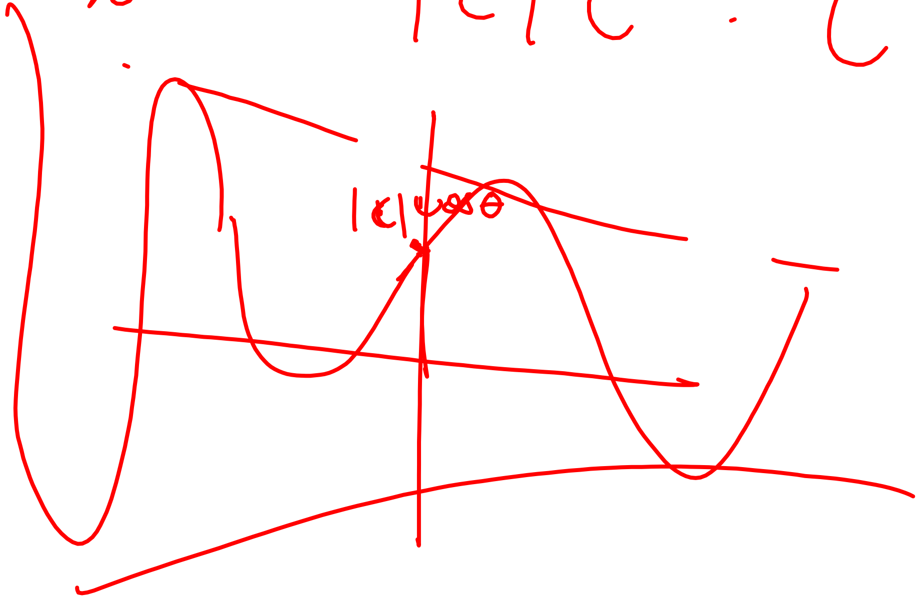
$\theta = \tan^{-1}\left(\frac{c_2}{c_1}\right)$

$a = \sigma + j\omega_0$

Generalised Exponential function $x(t) = Ce^{at}$

$$x(t) = |c| e^{j\theta} \cdot e^{\sigma t} \cdot e^{j\omega t}$$

$$x(t) = |c| e^{\sigma t} \cdot e^{j(\omega t + \theta)}$$



Signals: Elementary

Generalised Exponential function $x(t) = Ce^{at}$

Signals: Elementary

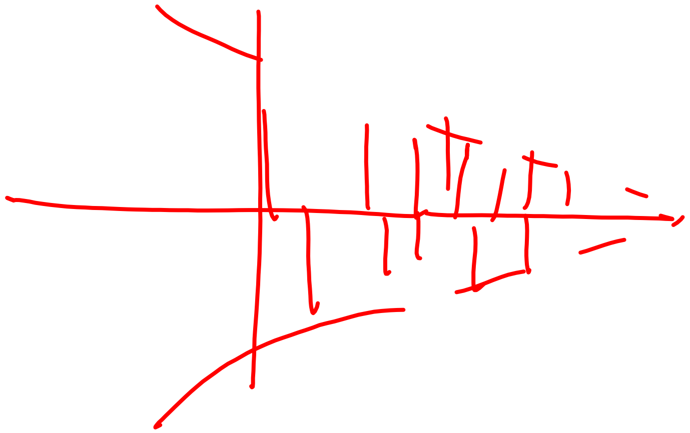
<u>Generalised Exponential function</u>

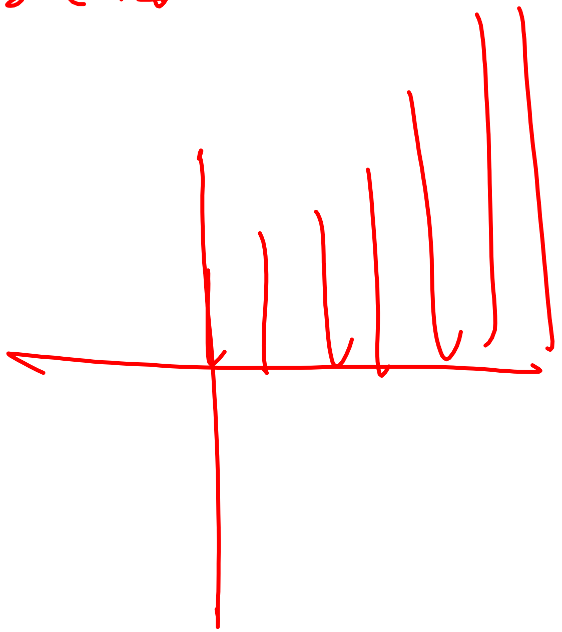
$$\boxed{x(n) = C\alpha^n} = Ce^{\beta n}$$

C= complex or real

α = complex or real

$x(n) = C\alpha^n = 5(-1/2)^n$





Signals: Elementary

<u>Generalised Exponential function</u> $$x(n) = C\alpha^n = Ce^{\beta n}$$

C= complex or real
α = complex or real

Signals: Elementary

Generalised Exponential function $x(n) = C\alpha^{n} = Ce^{\beta n}$

C= complex or real
α = complex or real

Signals: Elementary

Generalised Exponential function $x(n) = C\alpha^n = Ce^{\beta n}$

C= complex or real
α = complex or real

Signals: Elementary

<u>Generalised Exponential function</u> $x(n) = C\alpha^n = Ce^{\beta n}$

C= complex or real

α = complex or real

$$x(n) = Ce^{j\Omega_0 n} = C\left[\cos(\Omega_0 n) + j\sin(\Omega_0 n)\right]$$

$$\boxed{N_0 = \frac{2\pi}{\Omega_0} m}$$

$$x(n) = 5e^{j2\pi n} = 5e^{j2\pi n}$$

$$N_0 = \frac{2\pi}{2\pi} m$$

$$m = 1$$

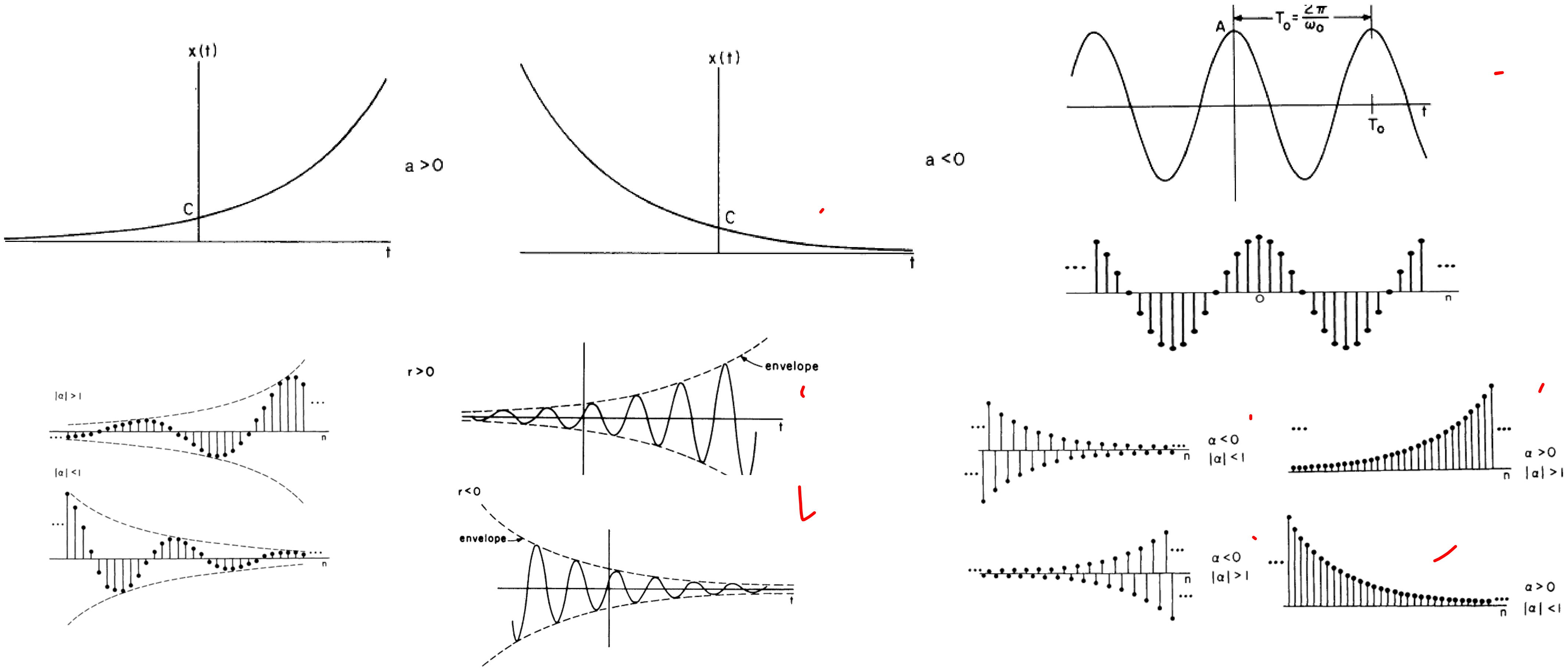
$$\boxed{N_0 = 1}$$

Signals: Elementary

<u>Generalised Exponential function</u>

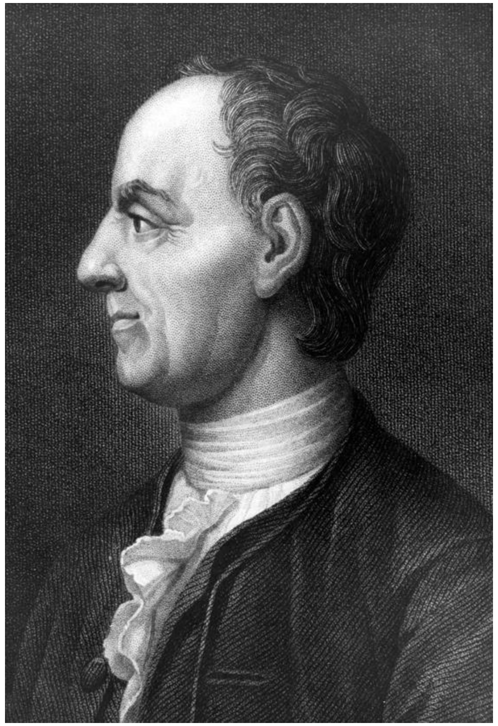
$$\boxed{x(t) = Ce^{at}}$$

$$\boxed{x(n) = C\alpha^{n}}$$



Leonhard Euler

$$e^{j\theta} = \cos(\theta) + jsin(\theta)$$



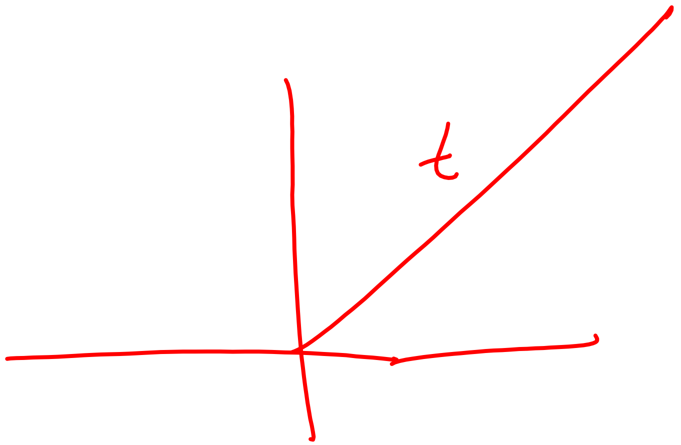
Leonhard Euler
Swiss mathematician
(1707-1783)

Test for:

- Periodicity
- Even Odd
- Energy, Power

Signals: Elementary

Unit ramp r(t)

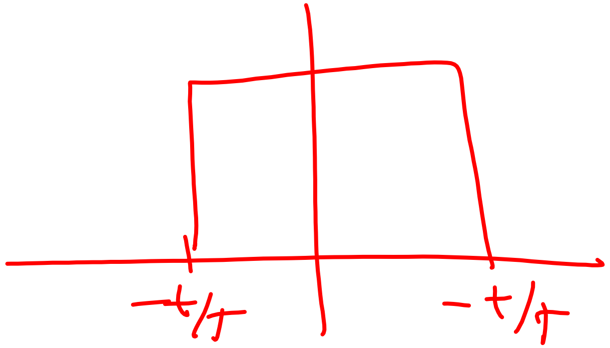


$$r(t) = \begin{cases} t & t > 0 \\ 0 & \text{else} \end{cases}$$

Test for:
- Periodicity
- Even Odd
- Energy, Power

Signals: Elementary

Gate function g(t)

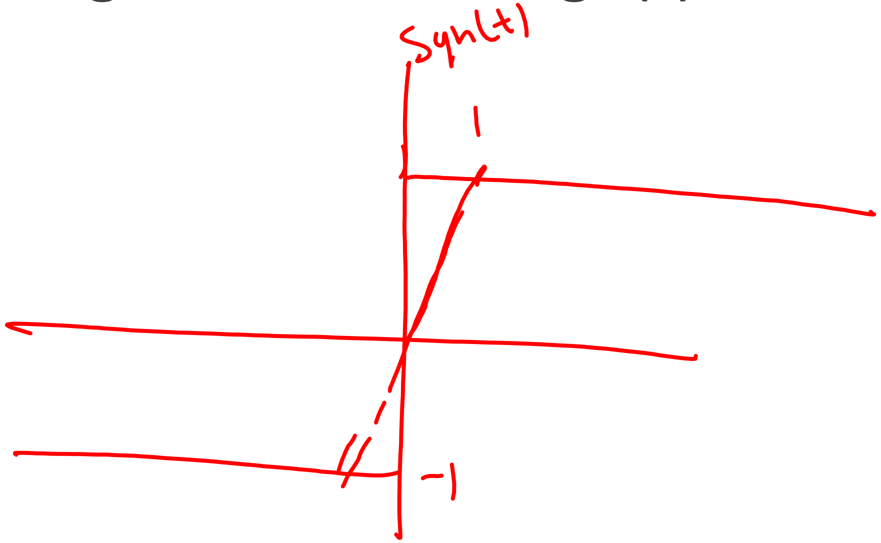


Test for:

- Periodicity
- Even Odd
- Energy, Power

Signals: Elementary

Signum function sgn(t)



$$= \begin{cases} 1 & t > 0 \\ -1 & t < 0 \end{cases}$$

$$\mathrm{sgn}(0) = 0$$

Exercises: Do at home

Homework

Chapter 1: *Alan V. Oppenheim*

Basic Problems

Queen Mary
University of London