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QHP5701 Exploratory Data Analysis

Periodicity & Fourier

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#Ref: Chapter 1, 3, 4, Oppenheim

Contents

- Periodicity — #Ref: Chapter 1, Oppenheim
- Fourier
  - Fourier Series — #Ref: Chapter 3, Oppenheim
  - Fourier Transform — #Ref: Chapter 4, Oppenheim

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Periodicity

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Note: Most of slides will be empty for in-class computations

Since we are following a text-book heavily, slides, will not include all the mathematical computations and details. Please check the text-book, for details.

#Ref: Chapter 1, Oppenheim

Periodicity

$e^{j\theta} = \cos(\theta) + j\sin\theta$

$= e^{j\omega_0(t+T)}$

- Condition: $x(t) = x(t + T)$

$$x(t) = e^{j\omega_o t}$$

$$e^{j\omega_0 t} = e^{j\omega_0 t} \cdot \underbrace{e^{j\omega_0 T}}_{=1}$$

$$\boxed{e^{j\omega_0 T} = 1}$$

$$e^{j\omega_0 T} = 1 = \cos(\omega_0 T) + j\sin(\omega_0 T)$$

$$\boxed{\omega_0 T = m2\pi}$$

$$m2\pi$$

$$\boxed{T_0 = \frac{2\pi}{\omega_0}}$$

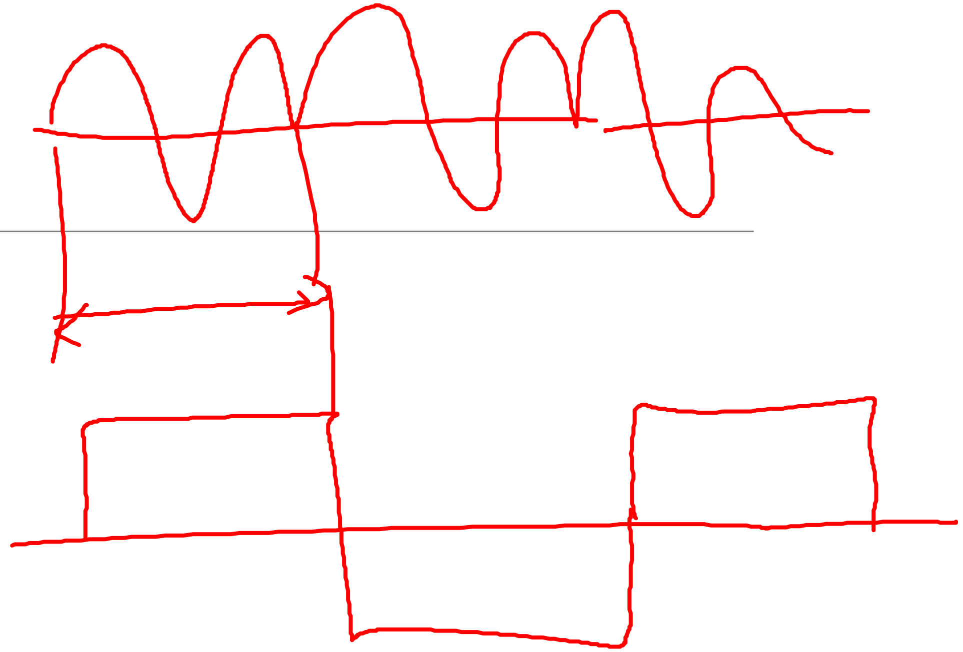
Periodicity

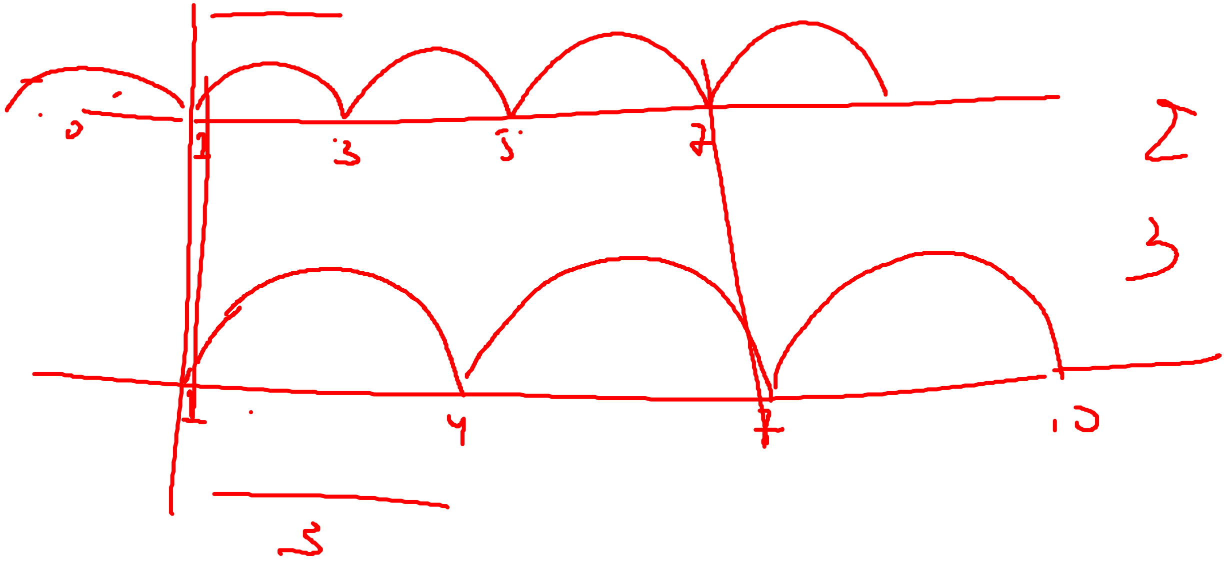
$$x(t) = x_1(t) + x_2(t) + ..$$

$$x(t) = 1 + 2\cos(10t + 1) - sin(4t - 1)$$

$T = ?$, T_1, T_2

$T = LCM(T_1, T_2)$





2 4 6 8 10

3 6

$$y(t) = \overset{T_1 = ?}{1} + \underline{2\cos(10t - 1)} - \underline{\sin(4t - 1)}$$

$\cancel{T = 2\pi}$

$T = \pi$

$$T_2 = \frac{2\pi}{10} = \frac{\pi}{5} \qquad T_3 = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$T = \mathrm{LCM}\left(\frac{\pi}{5}, \frac{\pi}{2}\right)$$

$\frac{\pi}{5}$, $\frac{2\pi}{5}$, ..., $\frac{10\pi}{5} = 2\pi$

$\frac{\pi}{2}$, ..., 2π

$\left(\frac{\pi}{5}, \frac{\pi}{3}\right) \Rightarrow \pi$

Periodicity

- Condition: $x(n) = x(n + N)$

$x(n) = e^{j\Omega_o n}$

$e^{j\Omega_o(n+N)} = e^{j\Omega_o n}$

$e^{j(\Omega_o+k2\pi)n} = e^{j\Omega_o n}$

$e^{j(\Omega_o + k2\pi)n} = e^{j\Omega_o n}$

$=$

$\Omega_o = 1\cancel{2}\pi$

$\Omega_o = 12\pi$

$e^{j\Omega_o N} = 1$

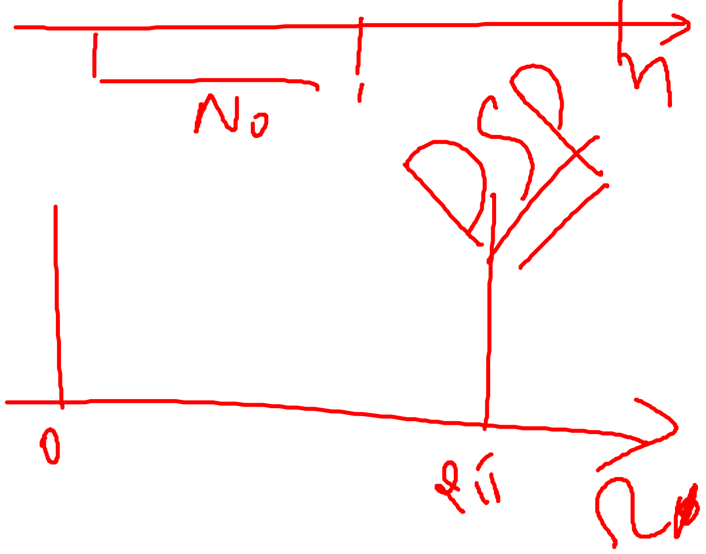
$\Omega_o N = m2\pi$

$\boxed{N_0 = m\frac{2\pi}{\Omega_o}}$

$e^{j\Omega_o n} = e^{j\Omega_o(n+N)}$

$= e^{j\Omega_o n} \cdot e^{j\Omega_o N}$

1



Periodicity

$$x(n) = x_1(n) + x_2(n) + ..$$

$$x(n) = e^{j\frac{2\pi}{3}n} + e^{j\frac{3\pi}{4}n}$$

$N_1 = m \frac{2\pi}{2\pi} \times 3 = 3$

$N_2 = \frac{2\pi \times 4}{3\pi} m = 8$ $m = 3$

$N_1 = 3$ $N_2 = 8$

$N = 24$

Periodicity

- Harmonically Related
  - Fundamental frequency f_0
  - m[th] harmonic $f_m = mf_0$

$$x(t) = \cos(10\pi t) + cos(20\pi t)$$

$x_1(t) \longrightarrow f_0 = 1\,kHz$

$x_2(t) \longrightarrow 2f_0 = 2\,kHz$

$3f_0$

Online Demos

Sinusoidal Player :

- https://nikeshbajaj.github.io/teaching/demos/SP/sinusoidal_player.html

Oscillator

- https://c4fa.github.io/nikJS/SineWave.html

Fourier Series

- https://www.falstad.com/fourier/

Contents

- Periodicity
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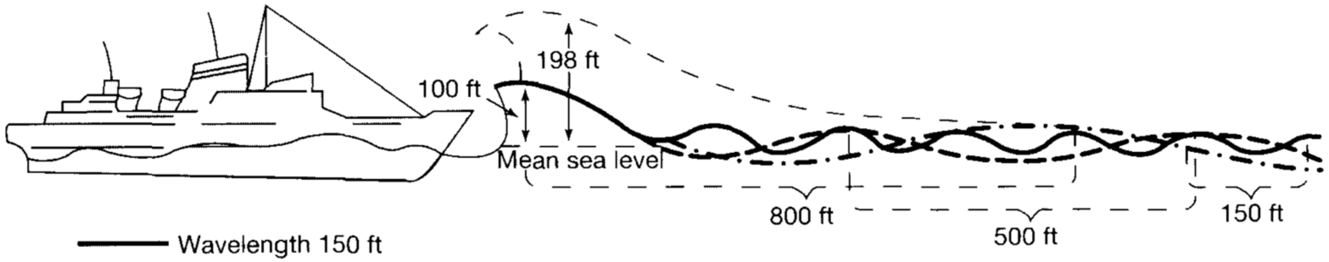
  - Fourier Series **#Ref: Chapter 3, Oppenheim**
  - Fourier Transform **#Ref: Chapter 4, Oppenheim**

QHP5701 Exploratory Data Analysis

Fourier: Analysis and Synthesis Tool



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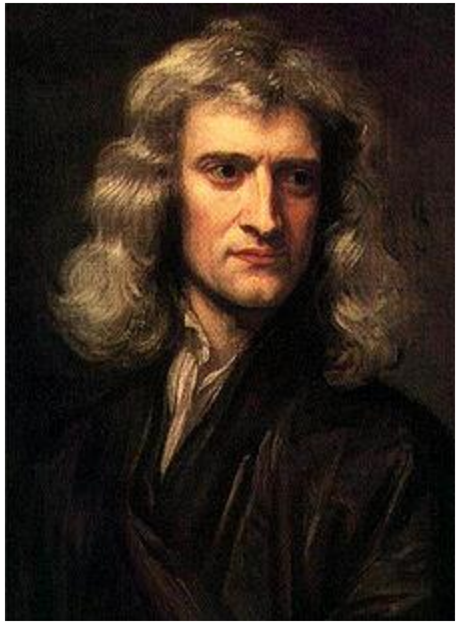
Guess….?

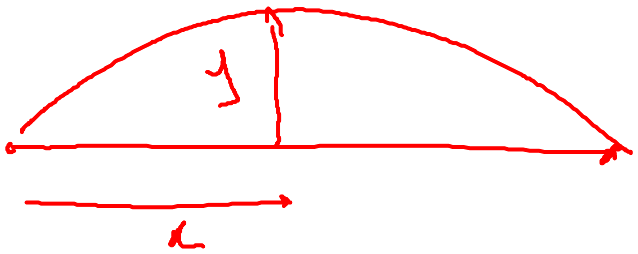


(4 January 1643 –
31 March 1727)

(1749-1827)

(1768 –1830)

History



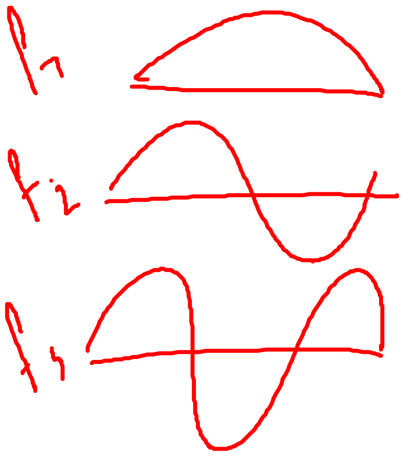
$y = f(x,t)$

In 1748

- **L. Euler**: Vibrating String
  - Vertical deflation at any time is the linear combination of ‘normal modes’, he showed it

In 1753

- **D. Bernouli** argued same on physical ground (No math) but was not accepted.
- idea of ‘Trigonometric series’ was discarded



In 1759

- **J. L. Langrange** strongly criticized Trig. Series.

"Fourier Series is a Great Mathematical Poem"
- Lord Kelvin (William Thompson)

Fourier

Jean Baptiste Joseph Fourier
21 March 1768 – 16 May 1830, Auxerre, France
Mathematician, Egyptologist, Revolutionary Discovery (1822). Scientific Advisor
Work: heat conduction

- He claimed that any periodic signal can be represented by a series of harmonically related sinusoidal.

FOURIER SERIES

- He also obtained a representation of Aperiodic signal, not as weighted sum of harmonically related sinusoidal, but as weighted Integral of sinusoidal that are not at all harmonically related.

FOURIER INTEGRAL or TRANSFORM

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Fourier Series

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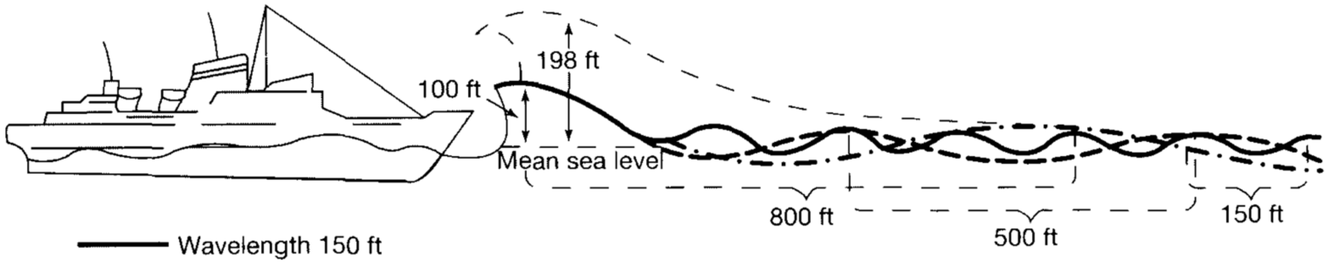


Any periodic signal can be represented by a series of harmonically related sinusoidal.

Fourier Series

$T_0 = \frac{2\pi}{\omega_0}$

$x_1(t)$

$a_{-2} e^{-2j\omega_0 t} + a_{-1} e^{-j\omega_0 t} + a_0 e^{j0t} + a_1 e^{j\omega_0 t} + a_2 e^{2j\omega_0 t} + a_3 e^{3j\omega_0 t}$

$T_0 = \frac{2\pi}{\omega_0}$

$x_2(t)$

$$\boxed{x(t) = \sum a_k e^{jk\omega_0 t}}$$

$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}}$$

Synth

$$\boxed{a_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} dt}$$

Analysis

Fourier Series

$$fund.period\ \boxed{T_0 = \frac{2\pi}{\omega_0}}$$

$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{jk\omega_0 t}}$$

- Synthesis equation

$$\boxed{\mathrm{a_k} = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jk\omega_0 t} dt}$$

- Analysis equation

$$\mathrm{a_k} = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jk\frac{2\pi}{T_0}t} dt$$

$$\underline{\mathrm{a_{-k}} = \mathrm{a_k^*}}$$

Fourier Series

$x(t) = \cos(3\pi t/4)$ $= \sum a_k e^{jk\omega_0 t}$

$x(t) = \sin(3\pi t/4)$

$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$

$T\omega_0 = \frac{3\pi}{3\pi}$

$\omega_0 = \frac{3\pi}{4}$

$\cos\left(\frac{3\pi}{4}t\right) = \frac{e^{+j\frac{3\pi}{4}t} + e^{-j\frac{3\pi}{4}t}}{2}$

$x(t) = \frac{1}{2} e^{j\frac{3\pi}{4}t} + \frac{1}{2} e^{-j\frac{3\pi}{4}t}$

$x(t) = \frac{1}{2} e^{j\omega_0 t} + \frac{1}{2} e^{-j\omega_0 t}$

$a_0 = 0$

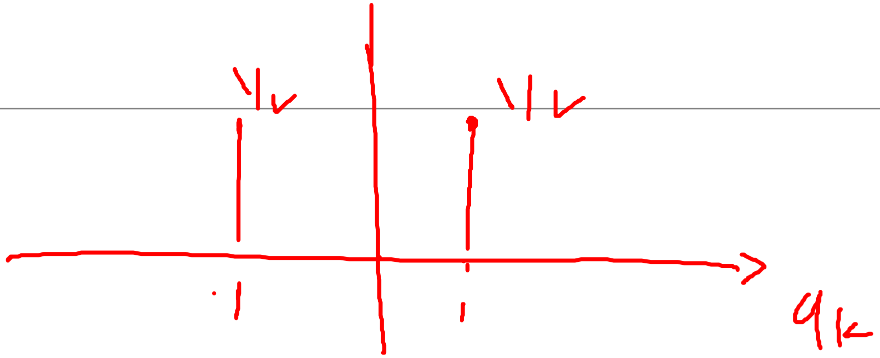
$a_1 = 1/2$

$a_{-1} = 1/2$

$a_k = 0$ $|k| > 1$

$a_0 = 0$

$a_1 = a_{-1} = \frac{1}{2}$



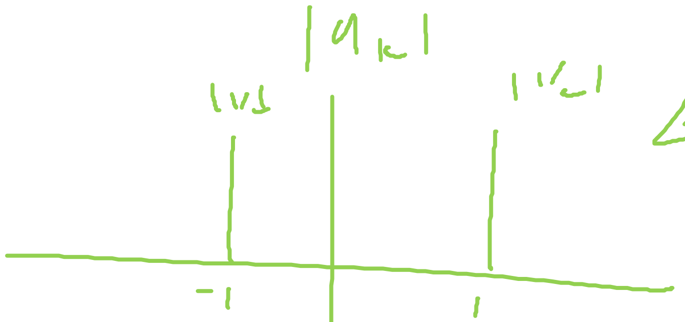
Line Spectrum

$a_0 = 0$

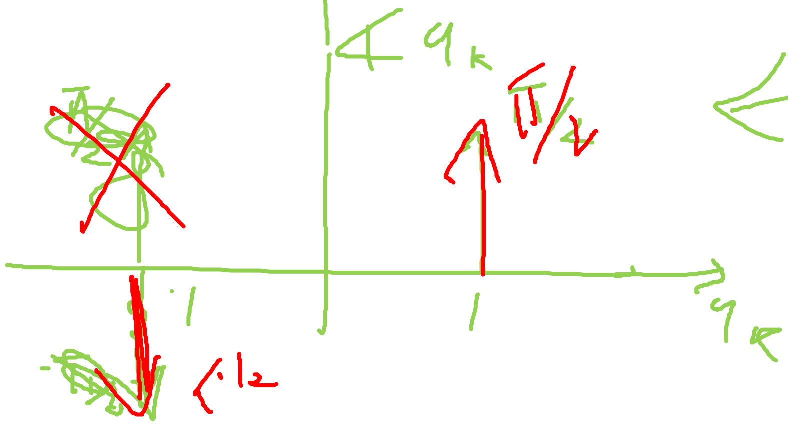
$a_1 = \frac{1}{2j}$ $a_{-1} = -\frac{1}{2j}$

$|a_1| = \frac{1}{\sqrt{2^2}} = \frac{1}{2}$

$\angle a_1 = \tan^{-1}(2/0) =$



Magnitude Spectrum



Phase Spectrum

Fourier Series

$x(t) = 1 + 2\cos(2\pi t) + \sin(3\pi t)$

$\cancel{\omega_0}$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

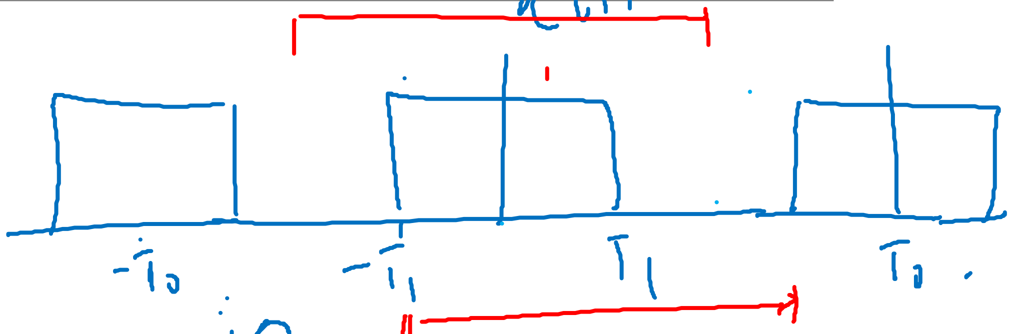
$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Fourier Series

$$x(t) = \begin{cases} 1 & for\ |t| < T_1 \\ 0 & for\ |t| > T_1 \end{cases} \quad with\ period\ T_0$$

$|t| < T_1$ $\qquad t < T_1$

$t < -T_1$



$$x(t) = \sum a_k e^{+jk\omega_0 t}$$

$$\boxed{\omega_0 = \frac{2\pi}{T_0}} =$$

$$a_0 = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-j0}\, dt = \frac{1}{T}\int_{T_0} x(t)\, dt$$

$$a_0 = \frac{1}{T_0}\int_{-T_1}^{T_1} 1\, dt = \frac{1}{T_0}\left[t\right]_{-T_1}^{T_1}$$

$$= \frac{1}{T_0}\left[T_1 + T_1\right] = \frac{2T_1}{1}$$

$$a_0 = \frac{2T_1}{T_0}$$

$$x(t) = \begin{cases} 1 & for\ |t| < T_1 \\ 0 & for\ |t| > T_1 \end{cases} \quad with\ period\ T_0$$

Fourier Series

$$\int e^{\alpha x} dx = \frac{e^{\alpha x}}{\alpha}$$

$$a_k = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{T_0}\int_{-T_1}^{T_1} 1\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T_0}\left(\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right]_{-T_1}^{T_1} = \frac{2 1}{k\omega_0 T_0}\left[\frac{-e^{-jk\omega_0 T_1} + e^{jk\omega_0 T_1}}{+j\cancel{k\omega_0} \times 2}\right]$$

$$a_k = \frac{2}{T_0\omega_0 k}\left[\frac{e^{jk\omega_0 T_1} - e^{-j\omega_0 T_1}}{2j}\right] = \frac{2}{k\omega_0 T}\sin(k\omega_0 T)$$

$T_1 = 1 \quad T_0 = 4 \Rightarrow \omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{2}$

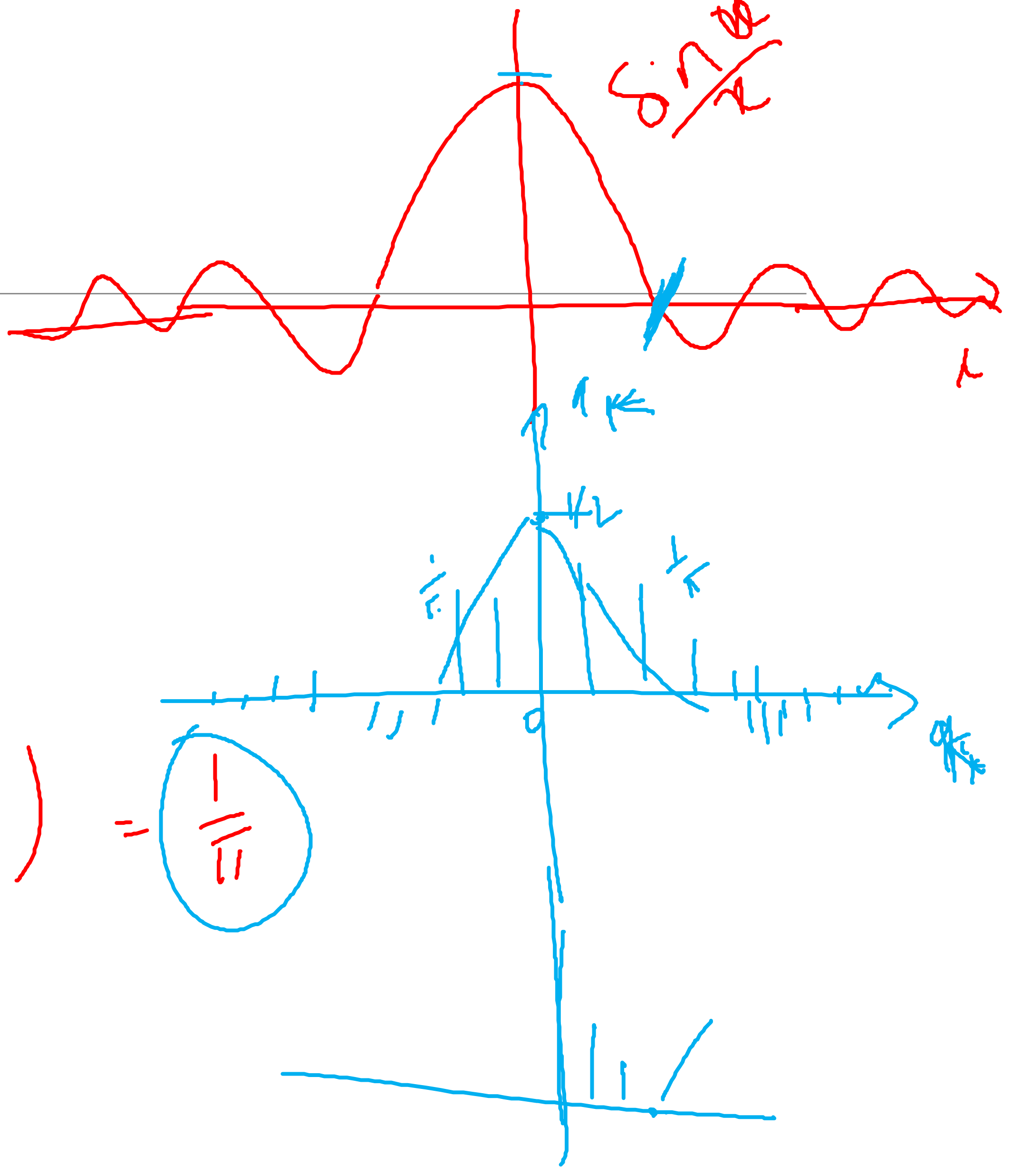
$$a_k = \frac{2}{k\omega_0 T_0} \sin(k\omega_0 T_1)$$

$$\boxed{a_k = \frac{1}{k\pi} \sin(k\omega_0 T_1)}$$

$$a_1 = \frac{1}{\pi} \sin(\omega_0 T_1) = \frac{1}{\pi} \sin\left(\frac{\pi}{2}\right) = \frac{1}{\pi}$$

$$a_{-1} = \frac{1}{\pi}$$

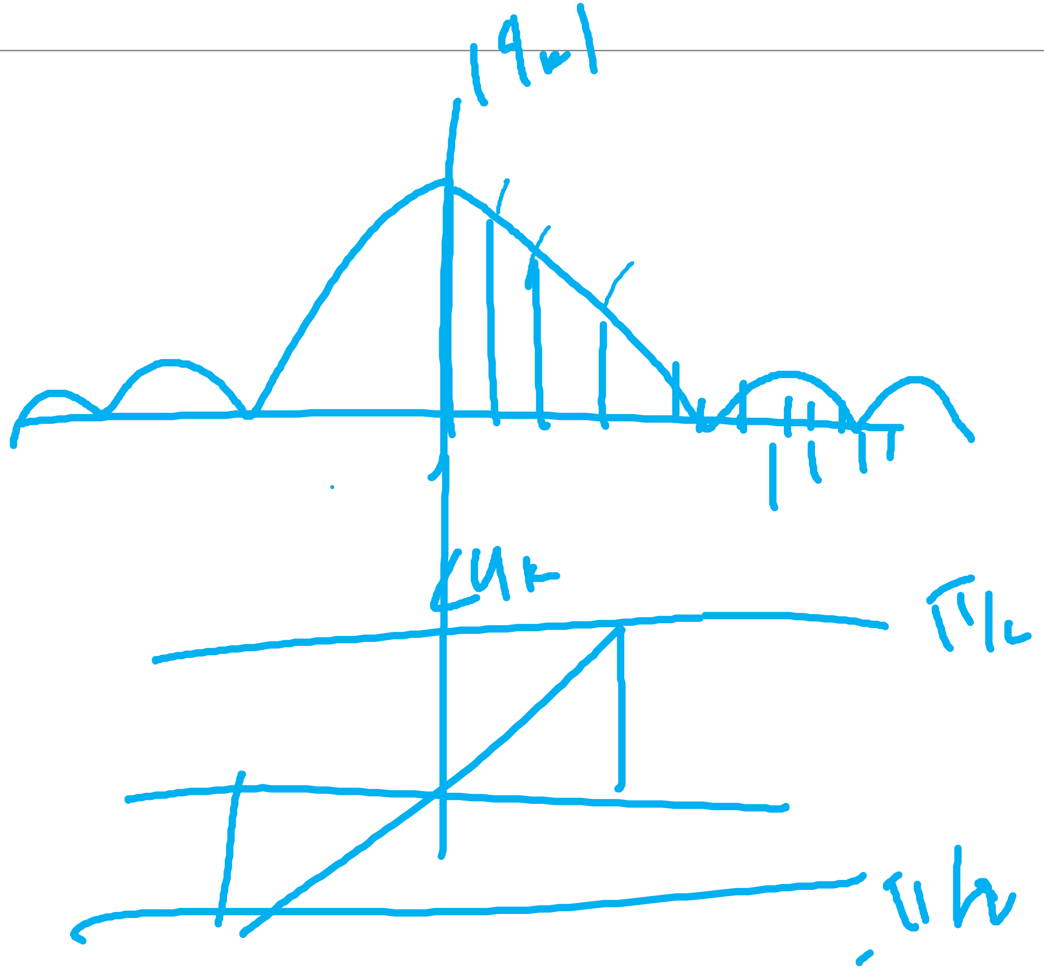
a_2



$$x(t) = \begin{cases} 1 & for\ |t| < T_1 \\ 0 & for\ |t| > T_1 \end{cases} \quad with\ period\ T_0$$

Fourier Series

$T_0 = 4T_1$



$$x(t) = \begin{cases} 1 \quad for\ |t| < T_1 \\ 0 \quad for\ |t| > T_1 \end{cases} \qquad with\ period\ T_0$$

Fourier Series

$T_0 = 8T_1$

$$x(t) = \begin{cases} 1 & for\ |t| < T_1 \\ 0 & for\ |t| > T_1 \end{cases} \qquad with\ period\ T_0$$

Fourier Series

$T_0 = 16T_1$

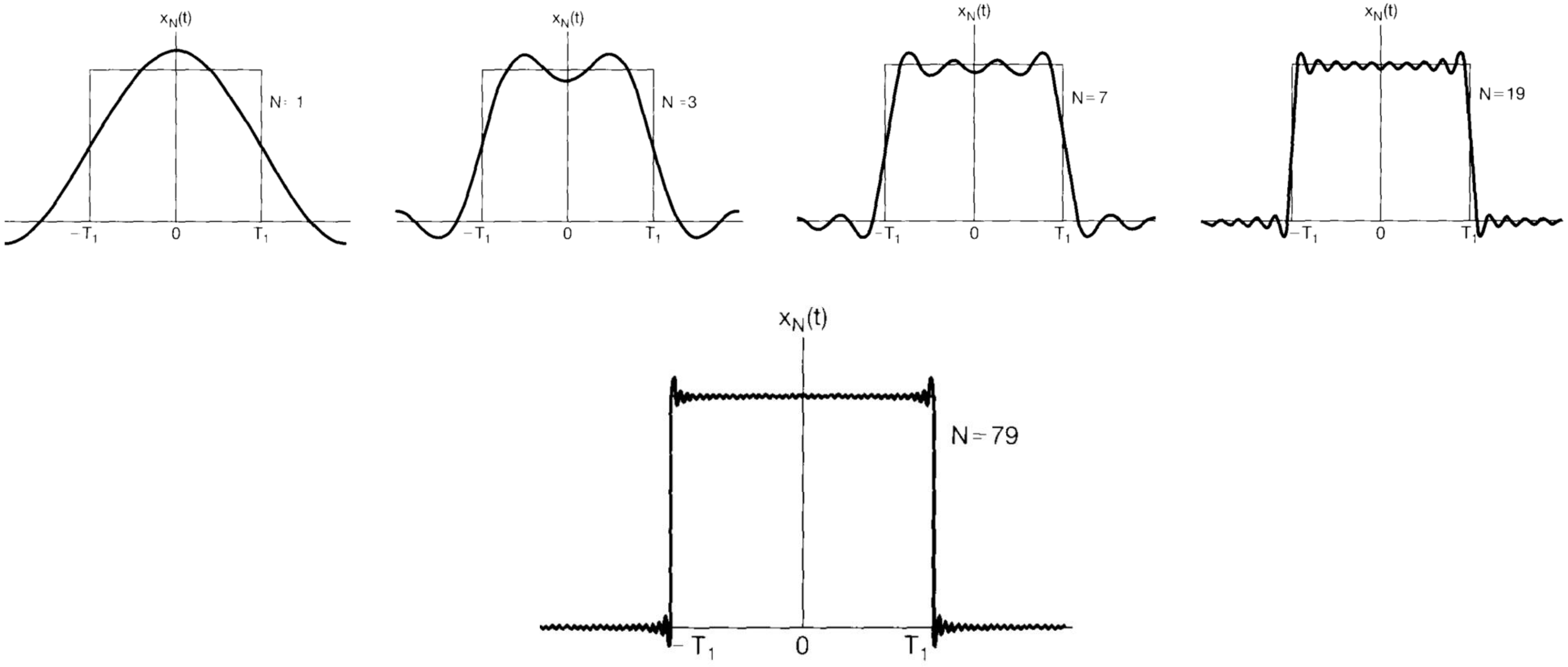
$$x(t) = \begin{cases} 1 & for\ |t| < T_1 \\ 0 & for\ |t| > T_1 \end{cases} \quad with\ period\ T_0$$

Fourier Series

- In general, $a_k's$ are complex
- Spectrum: Magnitude and Phase

Fourier Series: Synthesis

Synthesis with finite number of components



[Demo: https://www.falstad.com/fourier/](https://www.falstad.com/fourier/)

Fourier Series: Convergence

Dirichlet condition

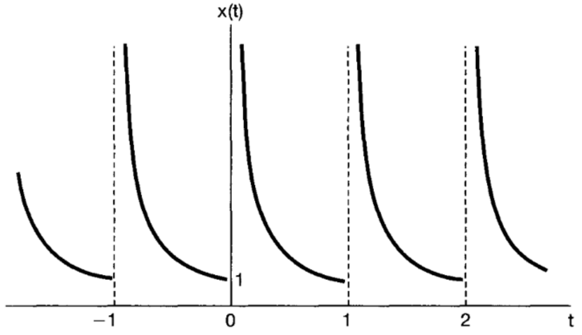
Developed by P. L. Dirichlet

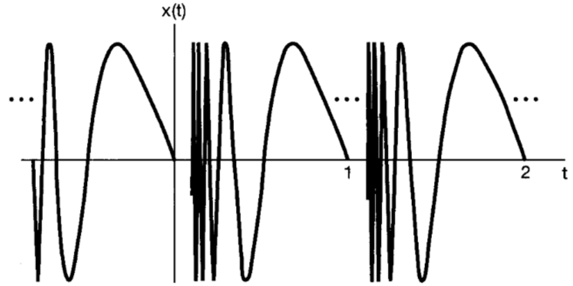
- <u>Condition 1</u>: *Over any period T, $x(t)$ must be absolutely integrable; that is*

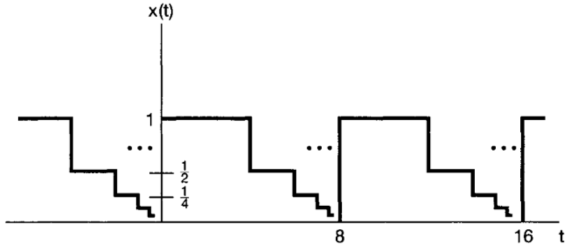
$$\int_T |x(t)|\,dt < \infty$$

- <u>Condition 2</u>: *In any finite interval of time, $x(t)$ is of bounded variation; that is, there are no more than a finite number of maxima and minima during any single period of the signal.*

- <u>Condition 3:</u> *In any finite interval of time, there are only a finite number of discontinuities. Furthermore, each of these discontinuities is finite.*







Fourier Series: Properties & Pairs

- Linearity
- Time- shifting
- Time Reversal
- Time-scaling
- Multiplication
- Parseval's Relation

$$x(t) = a\,\underbrace{x_1(t)} + b\,\underbrace{x_2(t)}$$

Table from Book

Chapter: 3

TABLE 3.1 PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Section	Periodic Signal	Fourier Series Coefficients
		$\left.\begin{array}{l} x(t) \\ y(t) \end{array}\right\}$ Periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$	a_k b_k
Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t} = e^{jM(2\pi/T)t} x(t)$	a_{k-M}
Conjugation	3.5.6	$x^*(t)$	a^*_{-k}
Time Reversal	3.5.3	$x(-t)$	a_{-k}
Time Scaling	3.5.4	$x(\alpha t), \alpha > 0$ (periodic with period T/α)	a_k
Periodic Convolution		$\int_T x(\tau)y(t-\tau)d\tau$	$Ta_k b_k$
Multiplication	3.5.5	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation		$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk\frac{2\pi}{T}a_k$
Integration		$\int_{-\infty}^{t} x(t)\,dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right)a_k = \left(\frac{1}{jk(2\pi/T)}\right)a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$\begin{cases} a_k = a^*_{-k} \\ \Re e\{a_k\} = \Re e\{a_{-k}\} \\ \Im m\{a_k\} = -\Im m\{a_{-k}\} \\ \lvert a_k\rvert = \lvert a_{-k}\rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals		$\begin{cases} x_e(t) = \mathcal{E}v\{x(t)\} & [x(t) \text{ real}] \\ x_o(t) = \mathcal{O}d\{x(t)\} & [x(t) \text{ real}] \end{cases}$	$\Re e\{a_k\}$ $j\Im m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

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QHP5701 Exploratory Data Analysis

Fourier Transform

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Any aperiodic signal can be represented as weighted Integral of sinusoidal that are not at all harmonically related.

Fourier Transform

$$x(t) = \int \underline{a_{\omega}}\, e^{j\omega_0 t}\, d\omega$$

$$x(t) = \frac{1}{2\pi} \int \underline{X(j\omega)}\, e^{j\omega t}\, d\omega$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

Fourier Transform

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$

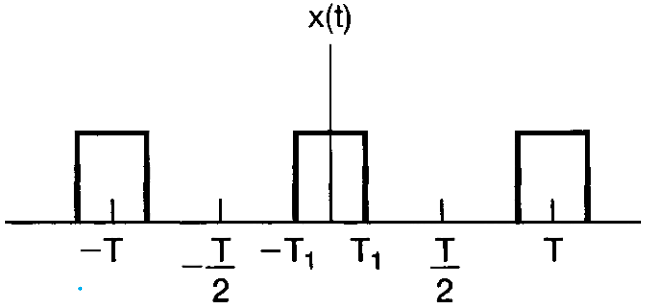
$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}dt$$

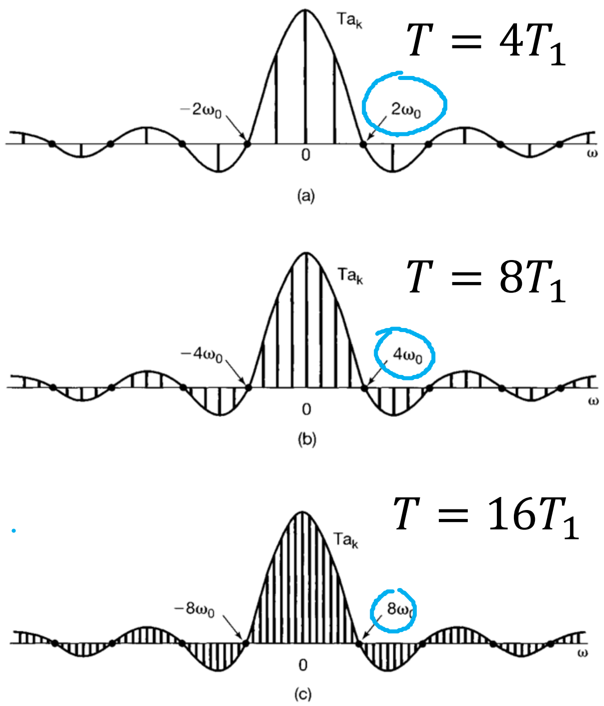
- Synthesis equation
- Analysis equation

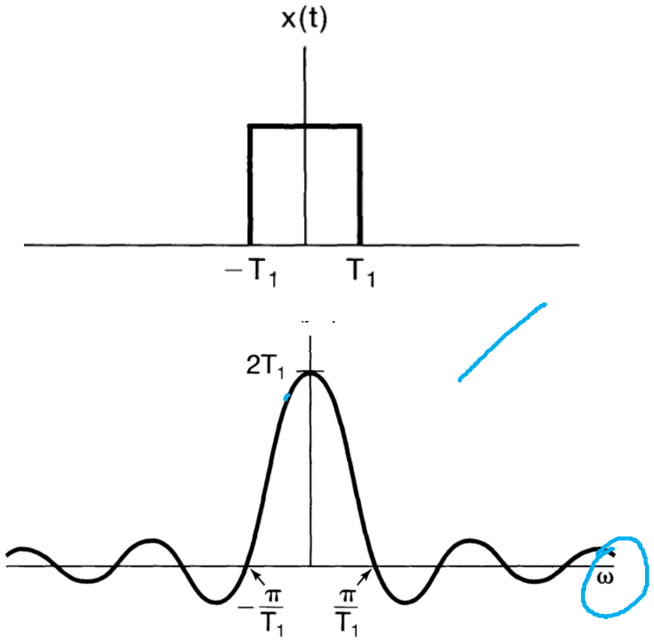
Fourier Transform

Fourier Series to Fourier Transform

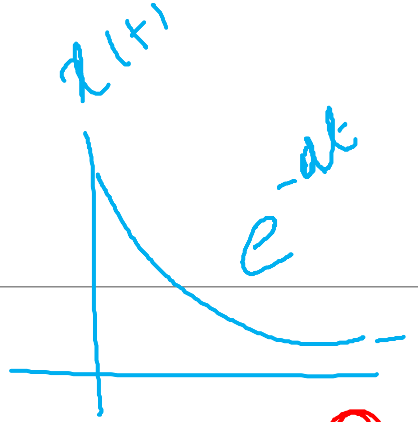
$T \rightarrow \infty$

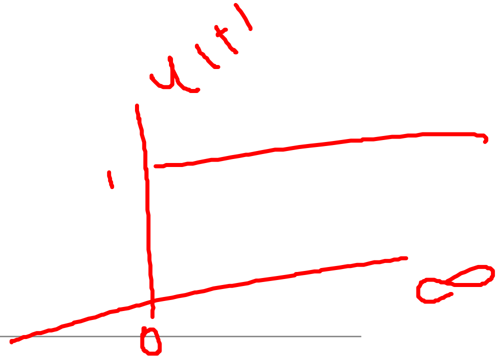






Fourier Transform





$x(t) = e^{-at}u(t)$ a>0

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = \int_{0}^{\infty} e^{-at}\, \cancel{u(t)}^{\,1}\, e^{-j\omega t}\, dt$$

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

$$= \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt = \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

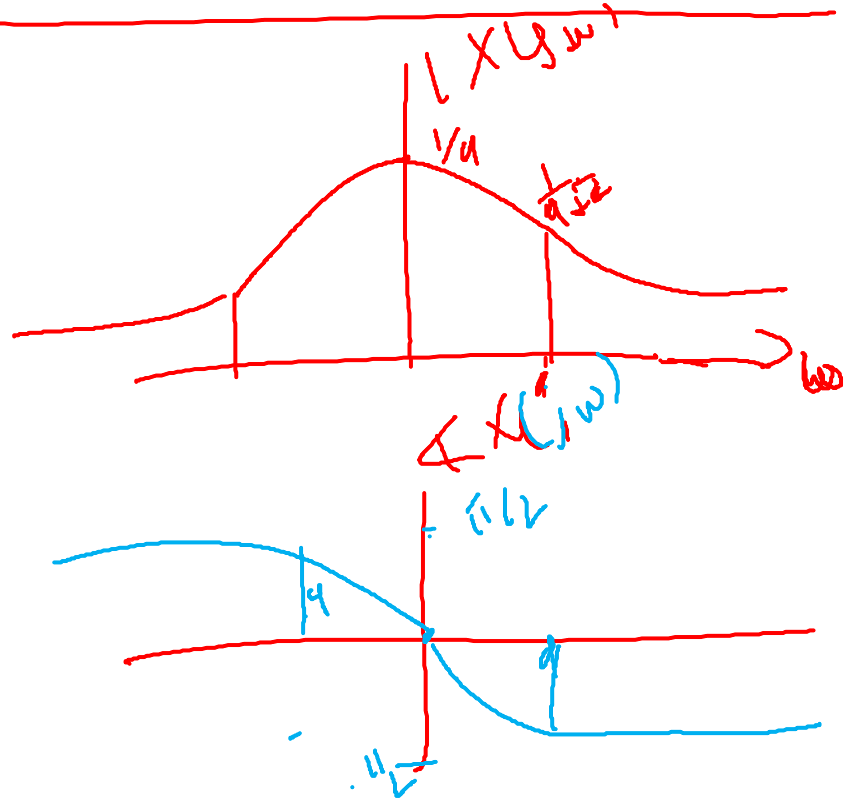
$$X(j\omega) = \int_0^\infty e^{-(a+j\omega)t}\,dt = \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right]_0^\infty$$

$$= \frac{1}{-(a+j\omega)}\left[0 - 1\right]$$

$$X(j\omega) = \frac{1}{a+j\omega}$$

$$\boxed{\angle X(j\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right)}$$

$$|X(j\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$$



Fourier Transform

$x(t) = e^{-a|t|}$ a>0

$e^{-at} u(t) + e^{at} u(-t)$

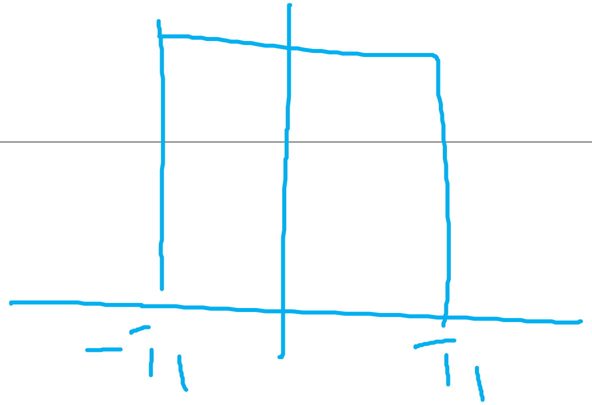
Fourier Transform

$x(t) = \delta(t)$

$$X(j\omega) = \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t}\, dt$$

$$= 1$$

Fourier Transform



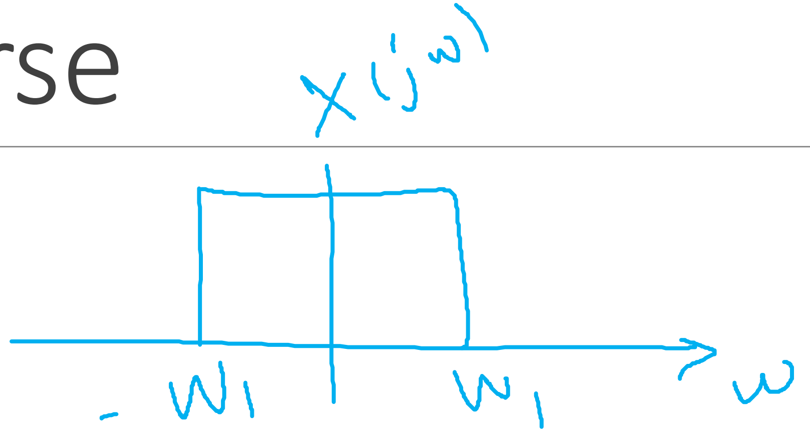
$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & |t| > T_1 \end{cases}$$

$$X(j\omega) = \int_{-T_1}^{T_1} e^{-j\omega t}\, dt = \frac{1}{-j\omega}\left[e^{-j\omega T_1} - e^{+j\omega T_1}\right]$$

$$= \frac{2}{\omega}\left(\frac{e^{j\omega T_1} - e^{-j\omega T_1}}{2j}\right) = \frac{2}{\omega}\sin(\omega T_1)$$

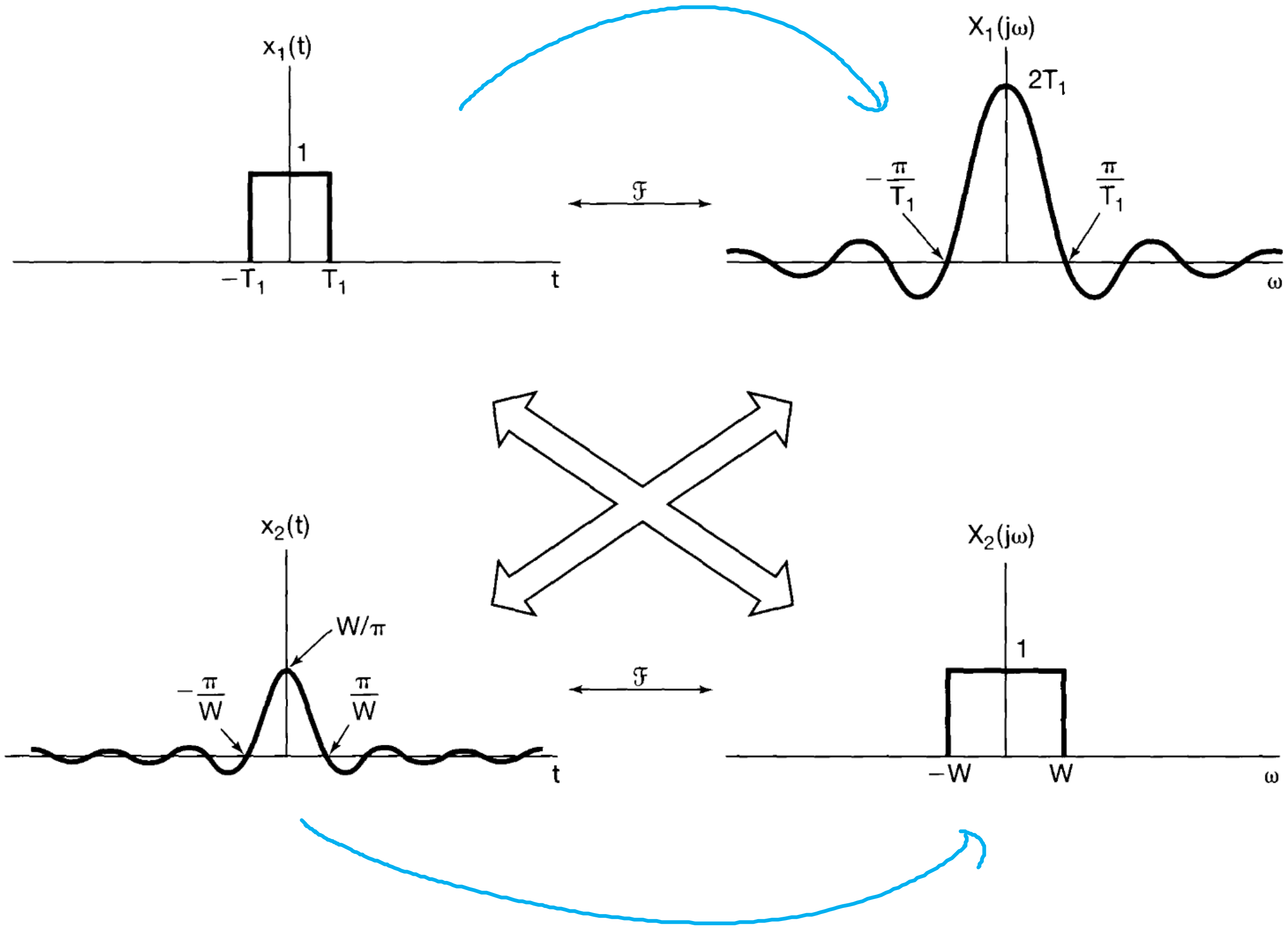
Fourier Transform: Inverse

$$X(j\omega) = \begin{cases} 1, & |\omega| < W_1 \\ 0, & |\omega| > W_1 \end{cases}$$



$$x(t) = \frac{1}{2\pi} \int_{-W_1}^{W_1} 1\, e^{j\omega t}\, d\omega$$

Fourier Transform: Duality



Fourier Transform: Convergence

Dirichlet condition

- Condition 1: $x(t)$ *must be absolutely integrable; that is*

$$\int_{-\infty}^{\infty} |x(t)|\, dt < \infty$$

- Condition 2: $x(t)$ *have finite number of maxima and minima for any finite interval of time.*

- Condition 3: $x(t)$ *have a finite number of discontinuities within any finite interval. Furthermore, each of these discontinuities must be finite.*

FT Properties

Chapter: 4

TABLE 4.1 PROPERTIES OF THE FOURIER TRANSFORM

Section	Property	Aperiodic signal	Fourier transform
		$x(t)$	$X(j\omega)$
		$y(t)$	$Y(j\omega)$
4.3.1	Linearity	$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$
4.3.2	Time Shifting	$x(t - t_0)$	$e^{-j\omega t_0}X(j\omega)$
4.3.6	Frequency Shifting	$e^{j\omega_0 t}x(t)$	$X(j(\omega - \omega_0))$
4.3.3	Conjugation	$x^*(t)$	$X^*(-j\omega)$
4.3.5	Time Reversal	$x(-t)$	$X(-j\omega)$
4.3.5	Time and Frequency Scaling	$x(at)$	$\frac{1}{\|a\|}X\left(\frac{j\omega}{a}\right)$
4.4	Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
4.5	Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\theta)Y(j(\omega-\theta))d\theta$
4.3.4	Differentiation in Time	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
4.3.4	Integration	$\int_{-\infty}^{t} x(t)dt$	$\frac{1}{j\omega}X(j\omega) + \pi X(0)\delta(\omega)$
4.3.6	Differentiation in Frequency	$tx(t)$	$j\frac{d}{d\omega}X(j\omega)$

4.3.7 Parseval's Relation for Aperiodic Signals

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega$$

FT Pairs

Chapter: 4

TABLE 4.2 BASIC FOURIER TRANSFORM PAIRS

Signal	Fourier transform
$\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
$\sin \omega_0 t$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
$x(t) = 1$	$2\pi\delta(\omega)$

TABLE 4.2 BASIC FOURIER TRANSFORM PAIRS

Signal	Fourier transform
$x(t)\begin{cases} 1, & \lvert t\rvert < T_1 \\ 0, & \lvert t\rvert > T_1 \end{cases}$	$\frac{2\sin\omega T_1}{\omega}$
$\frac{\sin Wt}{\pi t}$	$X(j\omega) = \begin{cases} 1, & \lvert\omega\rvert < W \\ 0, & \lvert\omega\rvert > W \end{cases}$
$\delta(t)$	1
$u(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$
$\delta(t - t_0)$	$e^{-j\omega t_0}$
$e^{-at}u(t), \mathcal{Re}\{a\} > 0$	$\frac{1}{a + j\omega}$
$te^{-at}u(t), \mathcal{Re}\{a\} > 0$	$\frac{1}{(a + j\omega)^2}$
$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$, $\mathcal{Re}\{a\} > 0$	$\frac{1}{(a + j\omega)^n}$

Fourier Transform: Properties

Linearity

$$x(t) = e^{-at}u(t) + 3\boxed{e^{-bt}u(t)} \quad \text{a, b>0}$$

$$X(j\omega) = \frac{1}{a + j\omega} + \frac{3}{(b + j\omega)}$$

Fourier Transform: Properties

Time-shift

$x(t) = e^{-a(t+1)}u(t+1)$ a>0

$e^{-at}u(t) \quad \frac{1}{a+jw}$

$X(jw) = \frac{1}{4+jw} e^{jw1} = \frac{e^{jw}}{4+jw}$

Fourier Transform: Properties

Frequency-shift

$$\mathrm{X}(j\omega) = \begin{cases} 1, & W_1 < |\omega| < 2W_1 \\ 0, & else. \end{cases}$$

Online Demos

Sinusoidal Player :

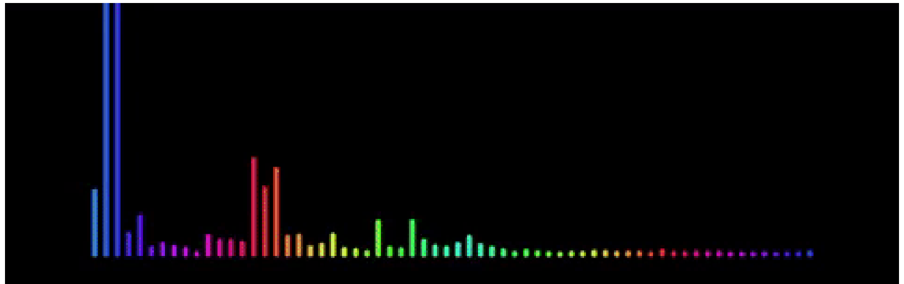
- https://nikeshbajaj.github.io/teaching/demos/SP/sinusoidal_player.html

Oscillator

- https://c4fa.github.io/nikJS/SineWave.html
- https://c4fa.github.io/nikJS/Choreography.html

Fourier Series

- https://www.falstad.com/fourier/
- https://www.desmos.com/calculator



Exercises: Do at home

Homework

Book: Alan V. Oppenheim

Chapter 3 : Fourier Series – only continues-time

Chapter 4: Fourier Transform – only continues-time

Examples

Basic Problems

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