Queen Mary School Hainan
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QHP5701 Exploratory Data Analysis

Systems

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#Ref: Chapter 1, 3, 4, Oppenheim

Contents

- Continues-time Systems
- Discreet-time Systems
- Properties
- Convolution, Filtering

#Ref: Chapter 1, Oppenheim

#Ref: Chapter 2, Oppenheim

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Note: Most of slides will be empty for in-class computations

Since we are following a text-book heavily, slides, will not include all the mathematical computations and details. Please check the text-book, for details.

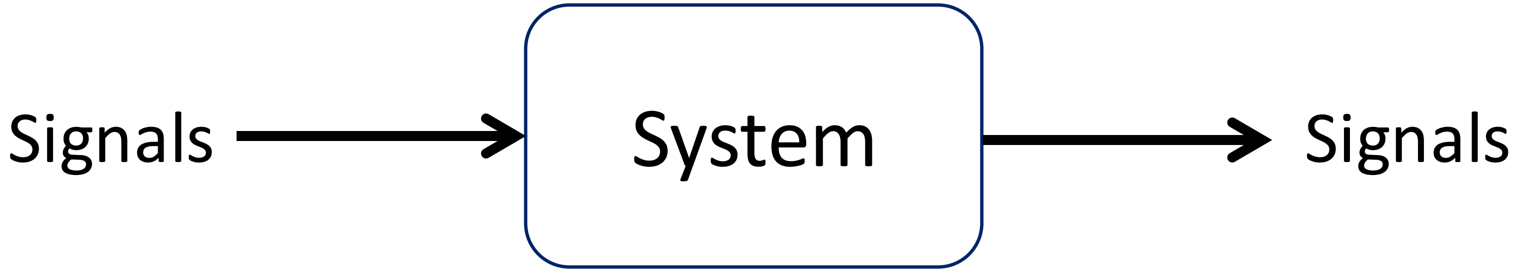
#Ref: Chapter 1, Oppenheim

Recall: Signals & Systems

- Signals and Systems?

  Any activity of human, can be thought of interaction or an interplay of the signals & systems

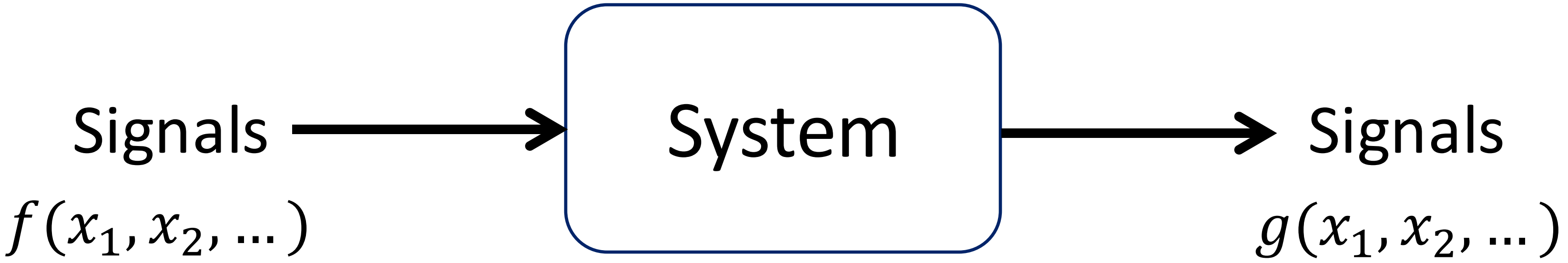
- A system as a block:
  - **is a meaningful** interconnection of physical devices or components
  - is an interconnection of subsystems, which are composed of physical devices or components
- A system by itself cannot achieve anything, it must be closely related to signals



Recall: Signals & Systems

- Signals and Systems?

  Any activity of human, can be thought of interaction or an interplay of the signals & systems



*A **signal**, in general, is a function of one or more independent variables*

System takes a signal, e.g. x(t), as input and produced more desirable output y(t)

e.g. Signal processing blocks

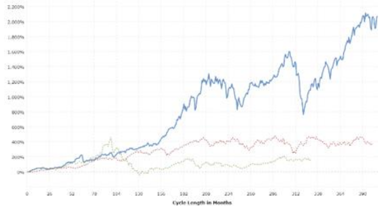
Signals

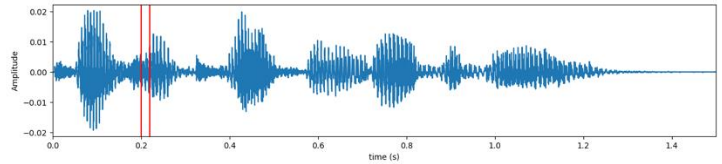
*A **signal**, in general, is a function of one or more independent variables*

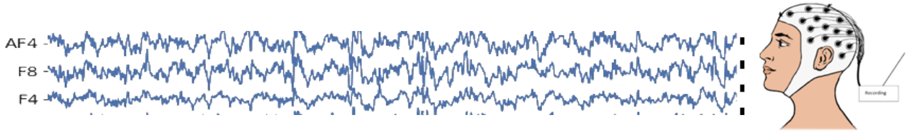
$f(x_1, x_2, \ldots), \quad x(t)$

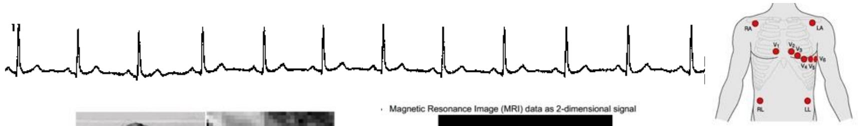
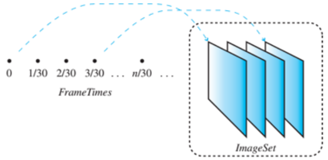
Examples

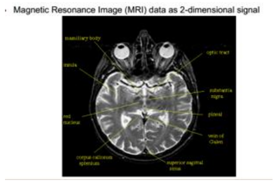
- Temperature in Hainan: x(t)
- Speech/audio: x(t)
- EEG, ECG, x(t)
- Image I(x,y)
- Video V(x,y,t)





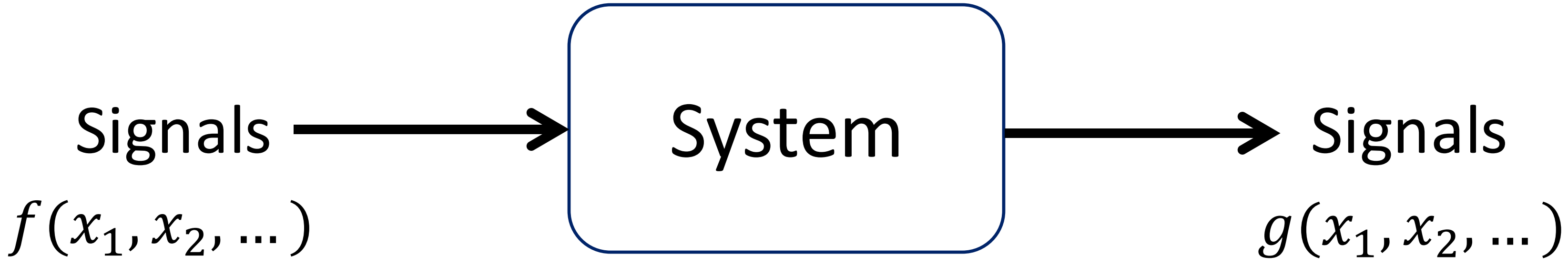
Signals

*A **signal**, in general, is a function of one or more independent variables*

$f(x_1, x_2, \ldots), \quad x(t)$

Anything that changes over one or more dimensions can be thought as a signal

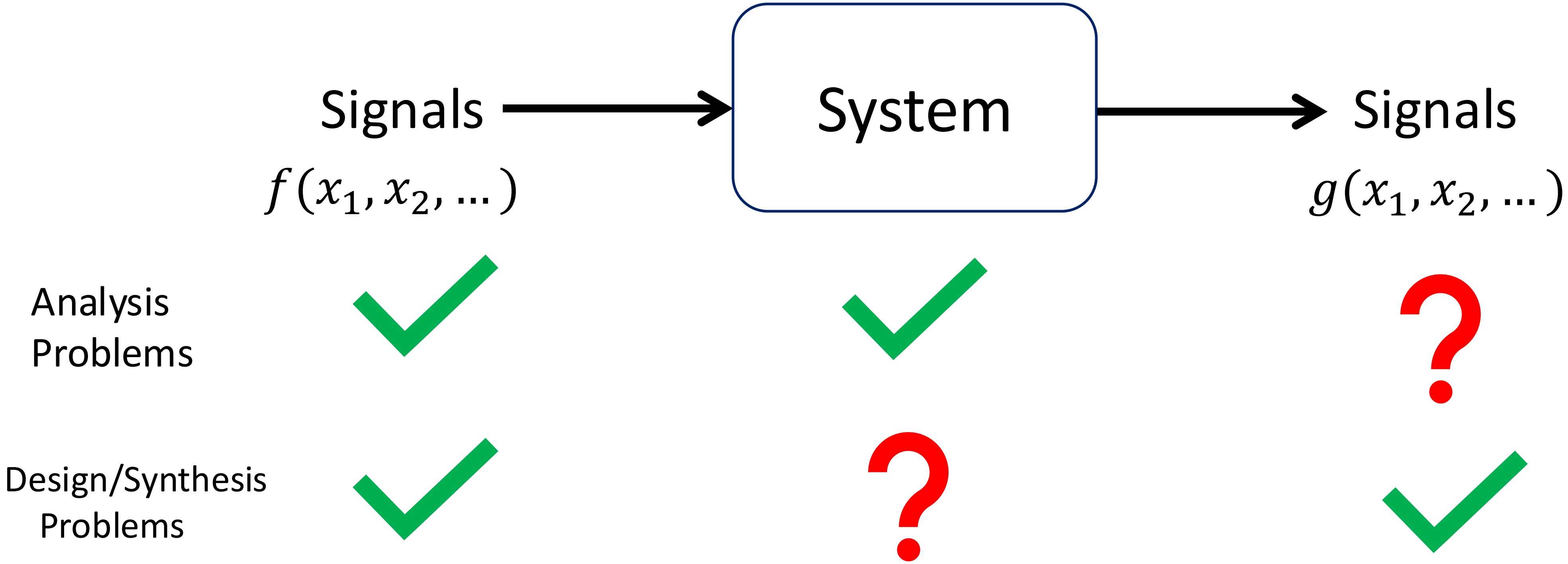
Examples of Signals & Systems



Examples

- Electrical Circuit
- Motor
- Heart Monitor
- …
- …

Signals & Systems: Problems



Analysis Problems: Given: Input signal and system Find: output signal

Design/Synthesis Problems: Given: Input signal and desired output signal Design: System

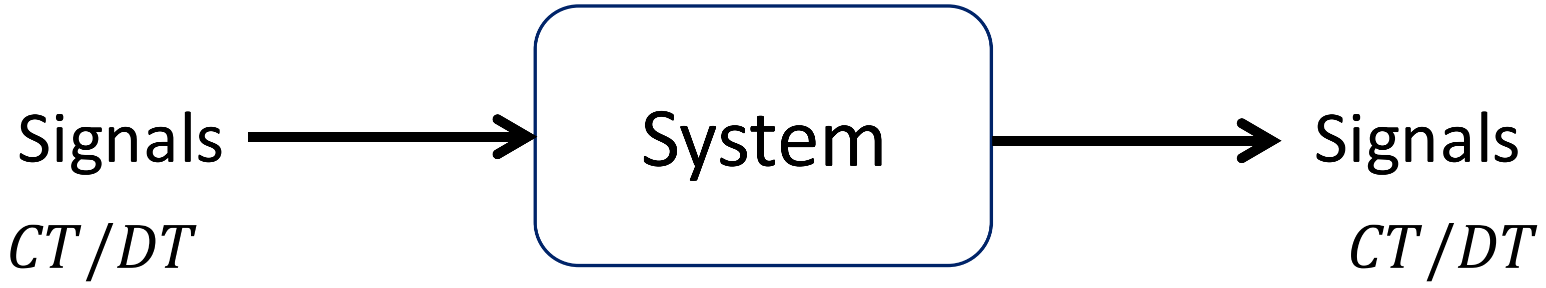
1. Input-output relationship
2. Block-diagram
3. Impulse response
4. Transfer function

Types of system

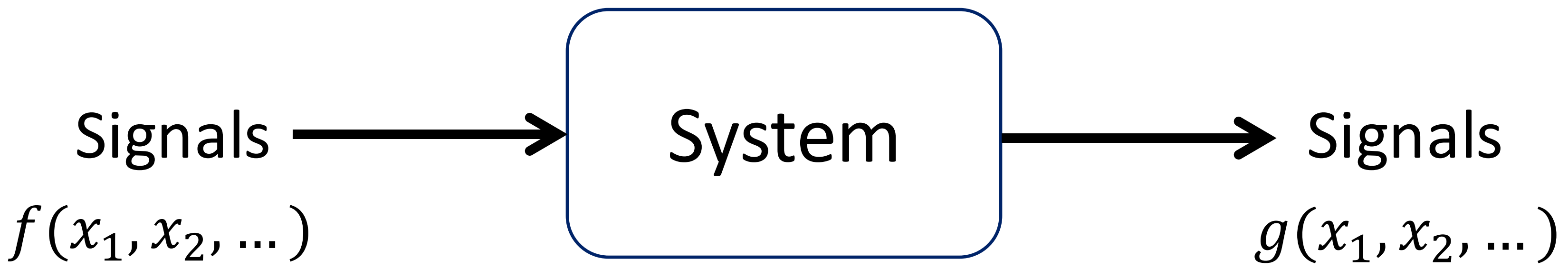
Continues-time System : CT System

Discreet-time System : DT System

Hybrid System (A/D and D/A Convertor)



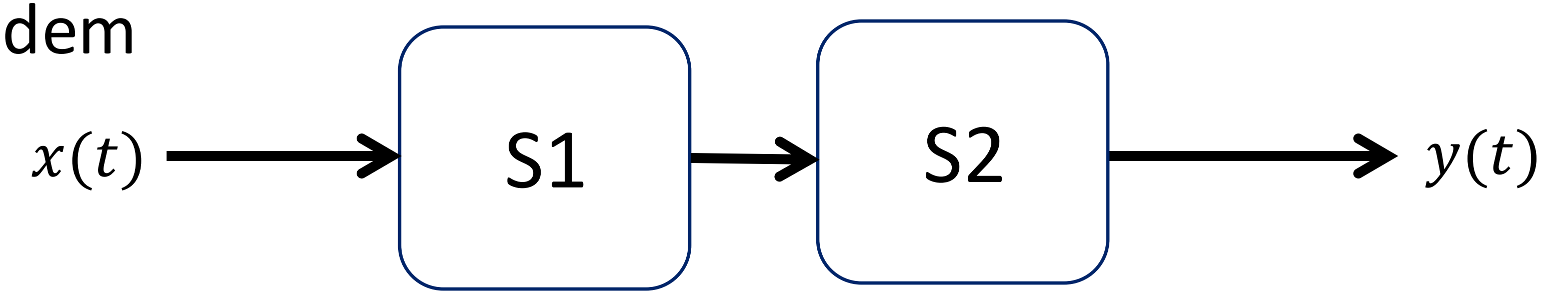
Types of system



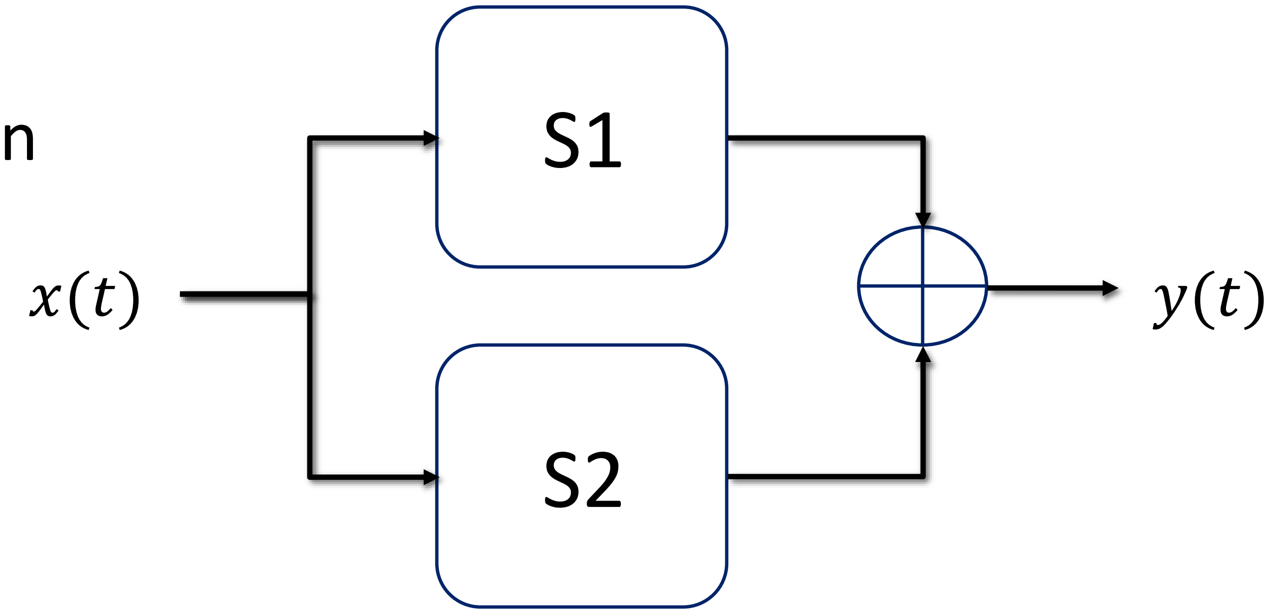
SISO – Single Input Single Output
MIMO- Multiple Input Multiple Output

Connection of systems (subsystems)

Cascade/Series/Tandem Connection

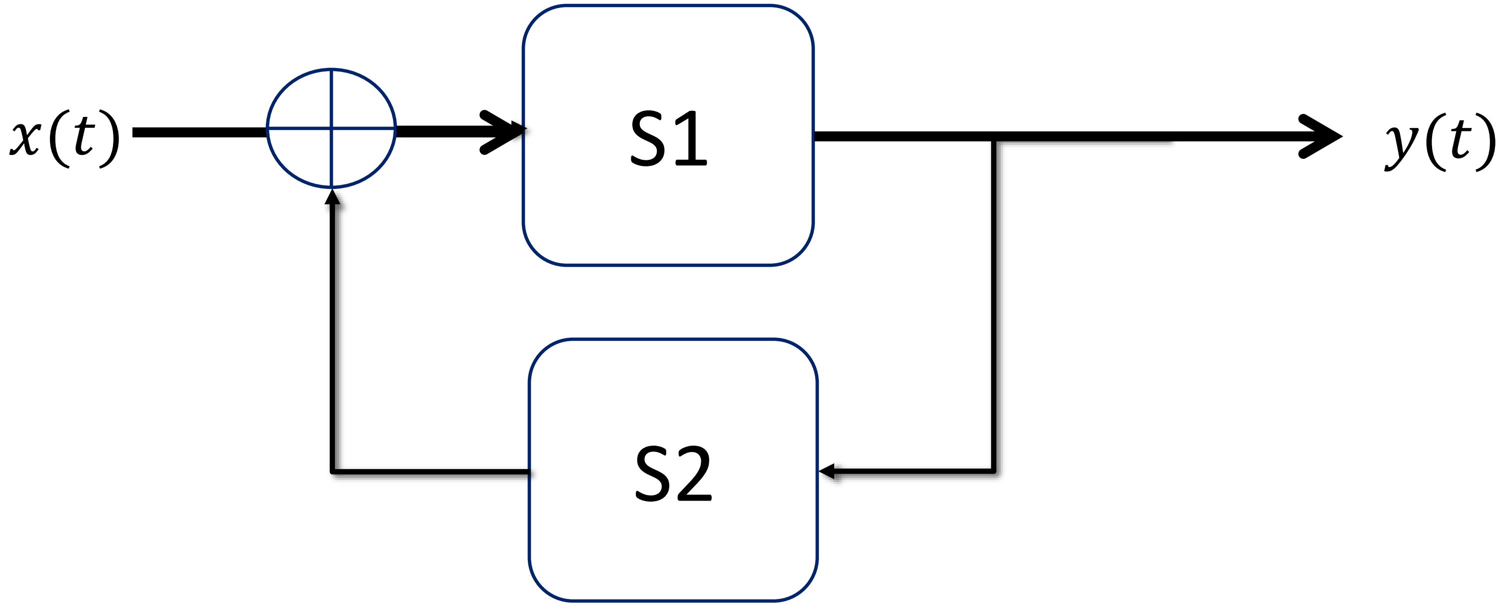


Parallel Connection



Connection of systems (subsystems)

Feedback Connection



Connection of systems (subsystems)

Hybrid: Mix

1. Input-output relationship
2. Block-diagram
3. Impulse response
4. Transfer function

Defining a system

System can be defined as a function f or a transformation

System can be defined in several ways.

1. Input-output relationship

$$x(t) \rightarrow y(t)$$

$$y(t) = T\{x(t)\}$$

$$y(t) = f(x(t))$$

$$\mathrm{y}(t) = x(t) + 0.5x(t-1)$$

$$\mathrm{y}(n) = x(n) + x(n-1)$$

$$\mathrm{y}(n) = Ax(n)$$

$$\mathrm{y}(t) = x(2t - 1)$$

Defining a system

System can be defined in several ways.

2. Block-diagram

$$y(n) = x(n) + x(n-1)$$

$$y(n) = Ax(n)$$

Defining a system

System can be defined in several ways.

3. Impulse response

4. Transfer function

Will see in due course

Properties of System

- With memory or Without Memory
- Linear or Nonlinear
- Time invariant or Time variant
- Stable or Unstable
- Causal or Non-causal
- Invertible or Non-invertible

Linear and Time Invariant System
LTI System

With memory or Without Memory

- Memory?
- Memoryless: Instantaneous
- With Memory: Dynamic System, initial conditions
- - Capacitor, Inductor

$$y(t) = x(t) + x(t-1)$$

$$y(t) = \left(x(t) + x^2(t)\right)^{1/2}$$

$$y(t) = x(t^2)$$

With memory or Without Memory

- Memory?

Invertible or Non-invertible

Invertible $x(t) \rightarrow y(t) \rightarrow x(t)$

$x(t) \rightarrow y(t)$

y$(t) \rightarrow x(t)$?

$$\mathrm{y(n)} = \sum_{k=-\infty}^{n} x(k)$$

$$y(t) = 0$$

$$y(t) = x^2(t)$$

Invertible or Non-invertible

Invertible $x(t) \rightarrow y(t) \rightarrow x(t)$

Causal or Non-causal

Causal: depends on past only

Causal – real-time, realizable system

Non-causal - can anticipate future

System S is causal iff

$$x_1(t) \equiv x_2(t), \qquad for\ t \le t_0$$

$$\Rightarrow y_1(t) \equiv y_2(t), \qquad for\ t \le t_0$$

Causal or Non-causal

System ***S*** is causal *iff*

$$x_1(t) \equiv x_2(t), \qquad for\ t \le t_0$$
$$\Rightarrow y_1(t) \equiv y_2(t), \qquad for\ t \le t_0$$

$$y(n) = \frac{1}{L}\sum_{k=-M}^{M} x(k) \qquad L = 2M + 1$$

$n < M, \qquad n > M$

Stability: Stable or Unstable

BIBO: Bounded input -> Bounded output

$$y(n) = \sum_{k=-\infty}^{n} x(k)$$

Time invariant or Time variant

$x(t) \rightarrow y(t)$

$\Rightarrow x(t - t_0) \rightarrow y(t - t_0)$

$y(n) = nx(n - n_0)$

$x_2(t) = x_1(t - T)$

$y_2(t) = y_1(t - T)$

$y_1(t) = \int_{-\infty}^{t} x_2(\tau)\, d\tau$

$y_2(t) = \int_{-\infty}^{t} x_1(\tau - T)\, d\tau$

$\tau - T = z$

$\tau = z + T$

$y_2(t) = \int_{-\infty}^{t-T} x(z)\, dz$

$= y_1(t - T)$

$|a+b| \le |a| + |b|$

$\alpha x_1 + \beta x_2$

Linear or Nonlinear

System S is linear iff, it satisfy

1) Superposition

$$x_1(t) \rightarrow y_1(t)$$
$$x_2(t) \rightarrow y_2(t)$$
$$x_1(t) + x_2(t) \rightarrow y_1(t) + y_2(t)$$

2) Homogeneity

$$\alpha x_1(t) \rightarrow \alpha y_1(t)$$

$$\alpha x_1(t) + \beta x_2(t) \rightarrow \alpha y_1(t) + \beta y_2(t)$$

input $x(t) = 0$ $y(t) = 0$

$$y(t) = |x(t)|$$

Non Lin

$$y(t) = mx(t) + c$$

$y(t) = |\alpha x_1 + \beta x_2|$

$= \alpha|x_1| + \beta|x_2|$

Linear or Nonlinear

$$= \alpha y_1(t) + \beta y_2(t)$$

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} \left(\alpha x_1(\tau) + \beta x_2(\tau)\right) d\tau$$

$$= \alpha \int^{t} x_1(t)\, d\tau + \beta \int_{-\infty}^{t} x_2(\tau)\, d\tau$$

$$y(t) = m x(t) + c$$

$$= m(\alpha x_1(t) + \beta x_2(t)) + c$$

$$= m\alpha x_1(t) + m\beta x_2(t) + c$$

$$\neq \alpha y_1(t) + \beta y_2(t)$$

Linear or Nonlinear

$$y(t) = \int_{-\infty}^{3t} x(\tau)d\tau$$

$$y = mx + b$$

Affine Sys

$$= \int \left(\alpha x_1(t) + \beta x_2(t)\right) d\tau$$

$$= \alpha \int_{-\infty}^{3t} x_1(\tau)\, d\tau + \beta \int_{-\infty}^{3t} x_2(\tau)\, d\tau = \alpha y_1 + \beta y_2$$

$$y(t) = \int_{-\infty}^{3t} x(\tau)d\tau$$

Memory/Memoryless?

Causal/Non-causal?

Time Invariant / Time variant?

Stable?

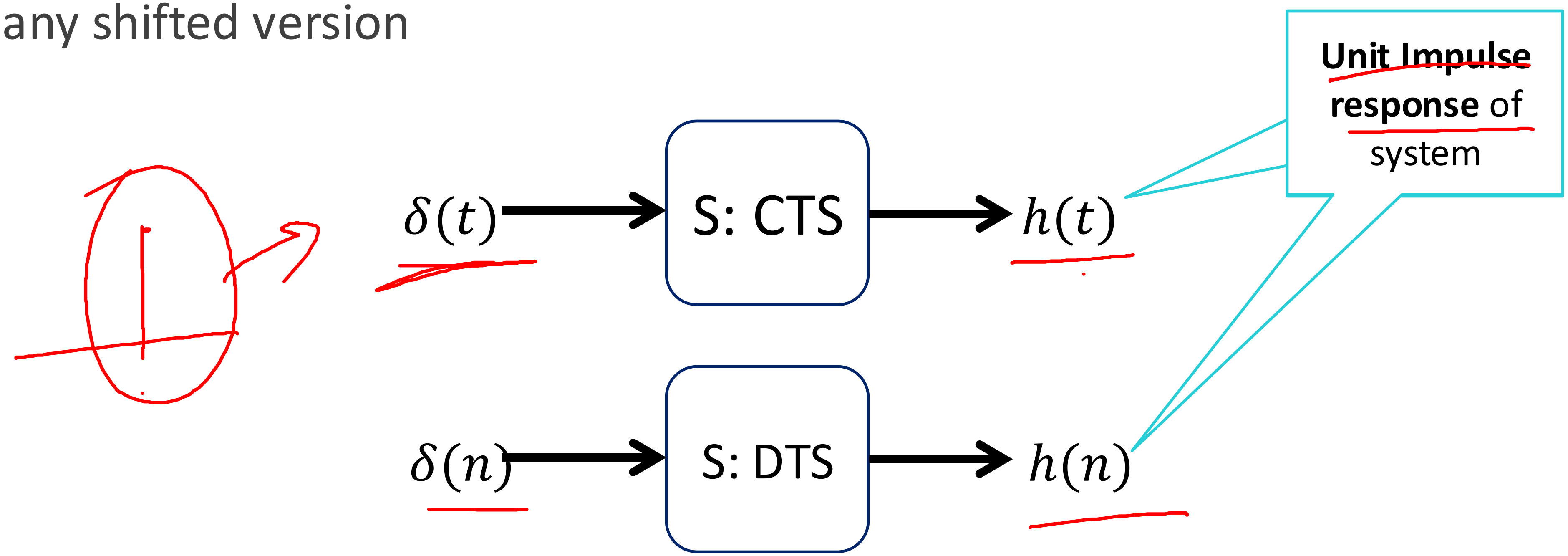
Invertible?

Linear Time Invariant System: LTI

Why Important?

Linearity: Decompose any arbitrary signal in elementary signals

TI: any shifted version



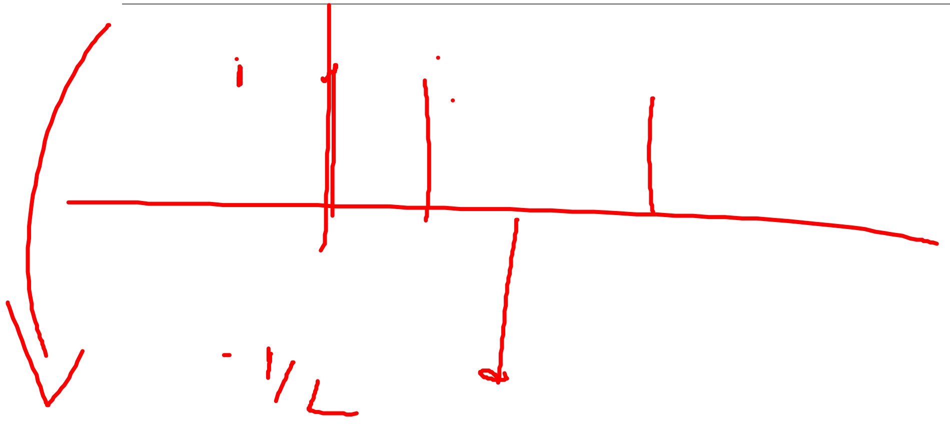
Linear Time Invariant System: LTI

If System is LTI, and **impulse response** is known, we can find the the output of system for any arbitrary input signal x(t)

Example:

$x(n) = \{1, 1, -\tfrac{1}{2}, 1\}$

$$h(n) = u(n)$$



$$x(n) = \delta(n) + \delta(n-1) - \frac{1}{2}\delta(n-2) + \delta(n-3)$$

$$y(n) = u(n) + u(n-1) - \frac{1}{2}u(n-2) + u(n-3)$$

Convolution sum

$$\delta(n) \longrightarrow h(n)$$

$$\delta(n-k) \longrightarrow h(n-k)$$

$$\sum x(k)\,\delta(n-k) \longrightarrow x(k)\,h(n-k)$$

$$x(n) \longrightarrow y(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$$

$$y(n) = x(n) * h(n)$$

Linear Time Invariant System: LTI

If System is LTI, and **impulse response** is known, we can find the the output of system for any arbitrary input signal x(t)

$$\delta(n) \rightarrow h(n)$$
$$\delta(n-k) \rightarrow h(n-k)$$
$$x(k)\delta(n-k) \rightarrow x(k)h(n-k)$$

$$\sum x(k)\delta(n-k) \rightarrow \sum x(k)h(n-k)$$
$$x(n) \rightarrow y(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

Convolution Summation

$$y(n) = x(n) * h(n)$$

Convolution

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k-r)h(n)$$

$$y(n) = x(n) * h(n)$$

$$y(n) = h(n) * x(n)$$

$$\begin{aligned} y(n) &= x(n) * h_1(n) * h_2(n) \\ &= h_1(n) * h_2(n) * x\,(n) \end{aligned}$$

Convolution: Example

Convolution: Example

Convolution: Example

Exercises: Do at home

Homework

Book: Alan V. Oppenheim

Chapter 1 : System Properties

Chapter 2: Convolution

Examples

Basic Problems

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