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QHP5701 Exploratory Data Analysis

Time Series Models

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#Ref: Chapter 1 to 4, +
Time Series Analysis by William WS Wei

Recap

So far, we have learned:

- Basic concepts of Stochastic Processes, Random Variables
- Time series as Stochastic Process
- Characteristics of time series: Stationarity, Seasonality, Non-stationarity

- Autocovariance Function AVF: γ_k and estimations
- Autocorrelation Function ACF: ρ_k and estimations

- Partial Autocorrelation Function PACF - will be covered after models

Random Walk

Model: $$X_n = X_{n-1} + Z_n$$ $$Z_n \sim N(\mu, \sigma^2)$$

$$X_n = \sum_{i=0}^{n} Z_i$$

$$Z_n \sim N(\mu, \sigma^2)$$

Random Walk

Model: $$X_n = X_{n-1} + Z_n$$

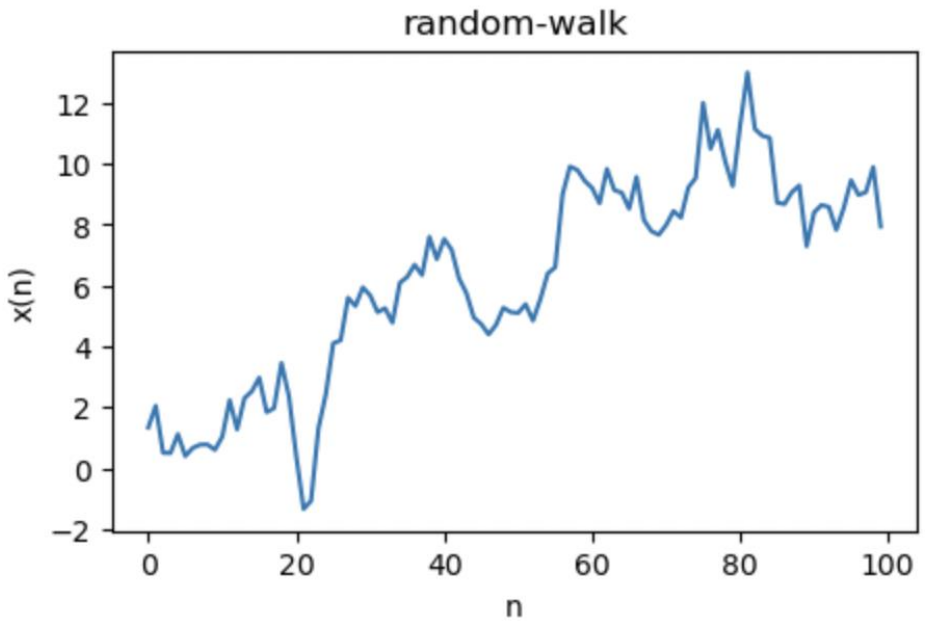
$$X_n = \sum_{i=0}^{n} Z_i$$

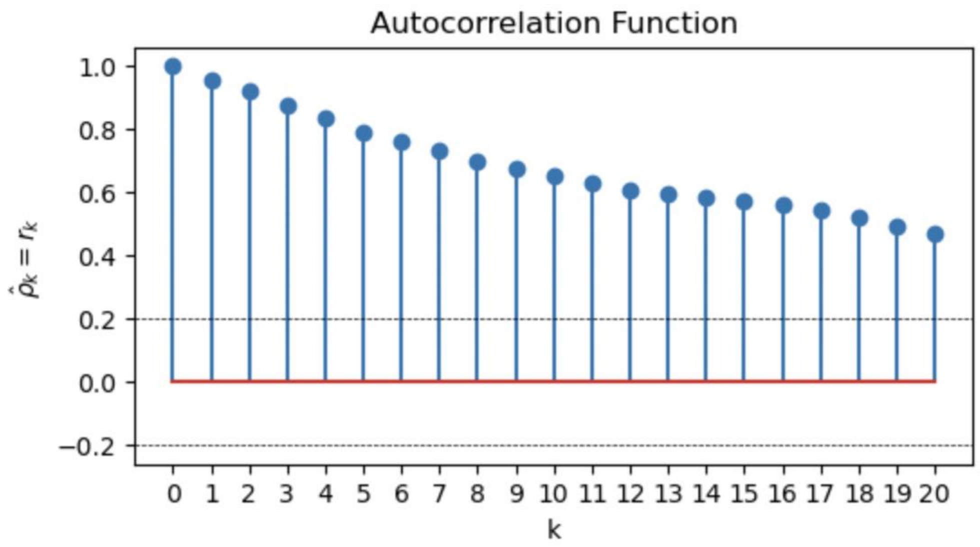
$$E[X_n] = E\left[\sum_{i=0}^{n} Z_i\right] = \sum_{i=0}^{n} E[Z_i] = \mu n$$

$$V[X_n] = V\left[\sum_{i=0}^{n} Z_i\right] = \sum_{i=0}^{n} V[Z_i] = \sigma^2 n$$

Stationary or not??

Random Walk: Simulate and compute AVF and ACF


random-walk
x(n)
n
12
10
8
6
4
2
0
−2
0
20
40
60
80
100


Autocorrelation Function
$\hat{\rho}_k = r_k$
k
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Difference Operator on random walk

Difference Operator

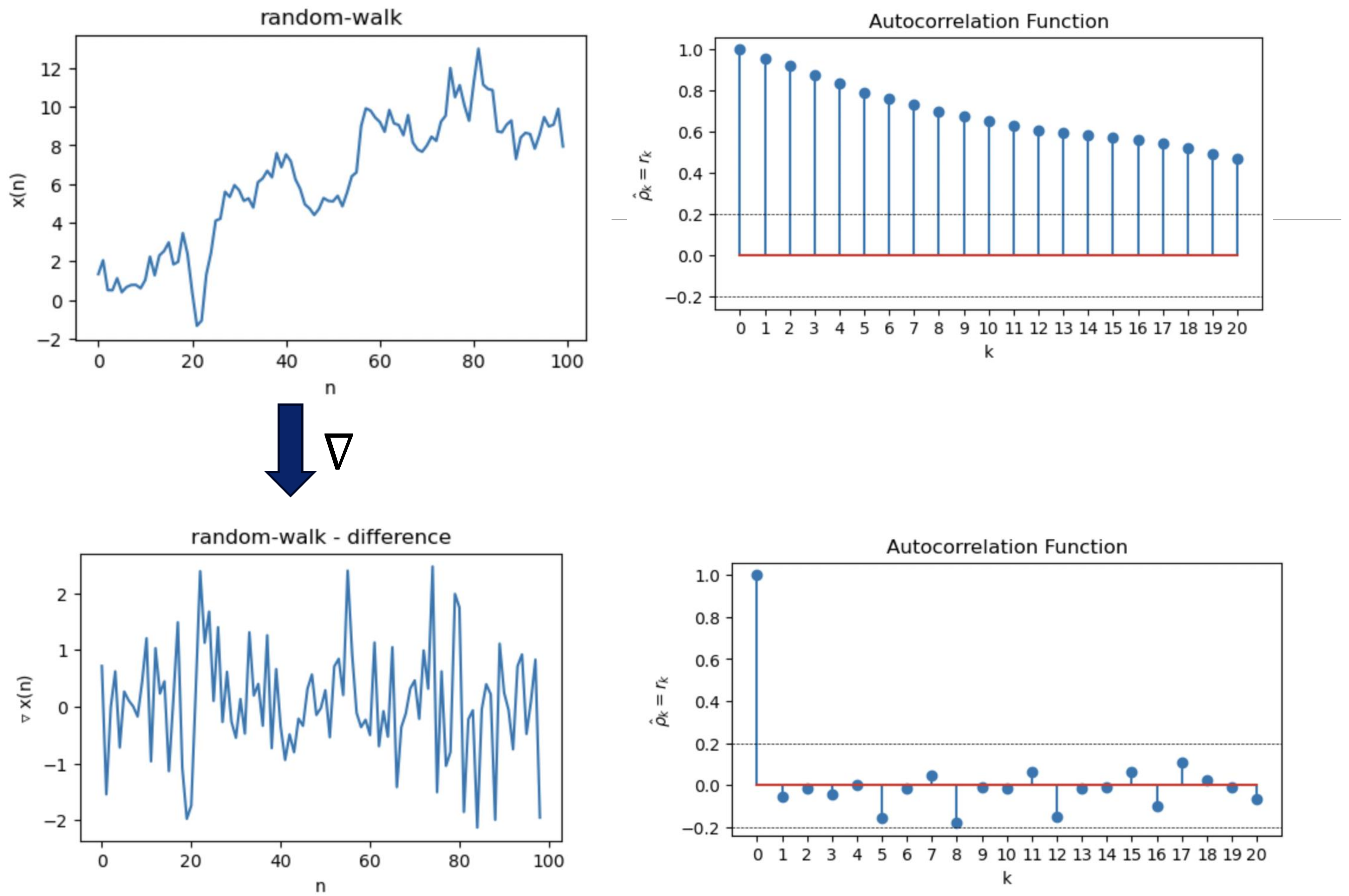
- Difference Operator $\nabla X = X_n - X_{n-1}$

- $X_n = X_{n-1} + Z_n$
- $X_n - X_{n-1} = Z_n = \nabla X$ noise

- Now we can compute AVF and ACF of ∇X

Stationary or not??

Simulate


random-walk
x(n)
n
Autocorrelation Function
$\hat{\rho}_k = r_k$
k
∇
random-walk - difference
∇ x(n)
n
Autocorrelation Function
$\hat{\rho}_k = r_k$
k

Connection to Signal and Systems

Moving Average processes: MA(q)

Stationary Time Series Models

$Z_n \sim N(\mu, \sigma^2)$ iid noise

Model: order q

$$X_n = Z_n + \theta_1 Z_{n-1} + \theta_2 Z_{n-2} \; \dots \; \theta_q Z_{n-q}$$

MA(1): $X_n = Z_n + \theta_1 Z_{n-1}$

MA(2): $X_n = Z_n + \theta_1 Z_{n-1} + \theta_2 Z_{n-2}$

Example

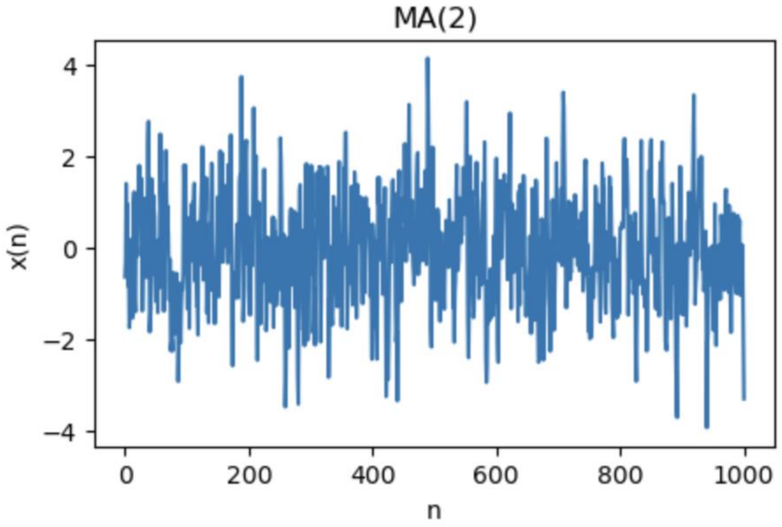
Simulate

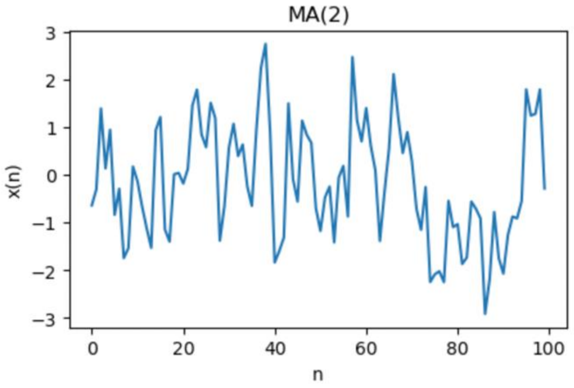
$$MA(2)$$
$$X_n = Z_n + 0.7Z_{n-1} + 0.2Z_{n-2}$$
$$Z_n \sim N(\mu = 0, \sigma^2 = 1)$$
$$Z_n \; are \; iid$$

Compute ACF

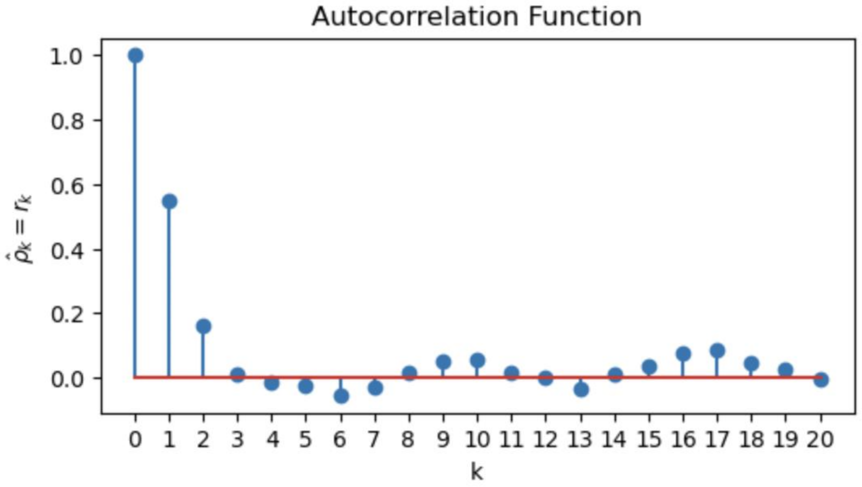
Simulate

$$MA(2)$$
$$X_n = Z_n + 0.7Z_{n-1} + 0.2Z_{n-2}$$
$$Z_n \sim N(\mu = 0, \sigma^2 = 1)$$
$$Z_n \ are \ iid$$





Compute ACF



Connection to Signal and Systems

Autoregressive Processes: AR(p)

Stationary Time Series Models

$Z_n \sim N(\mu, \sigma^2)$ iid noise

Model: order p

$$X_n = Z_n + history$$

$$X_n = Z_n + \phi_1 X_{n-1} + \phi_2 X_{n-2} \; \dots \; \phi_q X_{n-p}$$

AR(1): $X_n = Z_n + \phi_1 X_{n-1}$ -- random walk for ϕ_1=1

AR(2): $X_n = Z_n + \phi_1 X_{n-1} + \phi_2 X_{n-2}$

Example

Simulate

$$AR(2)$$
$$X_n = Z_n + 0.7X_{n-1} + 0.2X_{n-2}$$
$$Z_n \sim N(\mu = 0, \sigma^2 = 1)$$
$$Z_n \ are \ iid$$

Compute ACF

Connection to Signal and Systems

D: Delay *Operator (**B** backshift operator)*

There are different notations for this operator, we will use ***D*** for this operator taking from Signal and Systems lectures

$$DX_n = X_{n-1}$$
$$D^2X_n = X_{n-2}$$
$$D^pX_n = X_{n-p}$$

AR and MA

AR: $X_n = Z_n + \phi_1 X_{n-1} + \phi_2 X_{n-2} \ \dots \ \phi_q X_{n-p}$

MA: $X_n = Z_n + \theta_1 Z_{n-1} + \theta_2 Z_{n-2} \ \dots \ \theta_q Z_{n-q}$

$X_n = Z_n + \phi_1 D X_n + \phi_2 D^2 X_n \ \dots \ \phi_q D^p X_n$

$X_n(1 - (\phi_1 D + \phi_2 D^2 \ \dots + \phi_q D^p)) = Z_n$

$X_n f(D) = Z_n$

$X_n = \frac{1}{1 - (\phi_1 D + \phi_2 D^2 \ \dots \ + \phi_q D^p)} Z_n$

Example

ARMA

ARMA

ARMA

ARMA

ARIMA

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