

# Introduction to Statistics

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### Overview

### Session 1

- Describe your data
  - Descriptive statistics summarising the data
  - Visualisation (plots and figures)
- Inferential analysis:
  - Inference about population from sample
- Given two groups of data
  - Test the differences between groups (hypothesis testing)
  - Test the relationship between two variables (correlation)

Session 2 Lab Practice using SPSS

# Introduction to Statistics

## Part 1: Describe your Data

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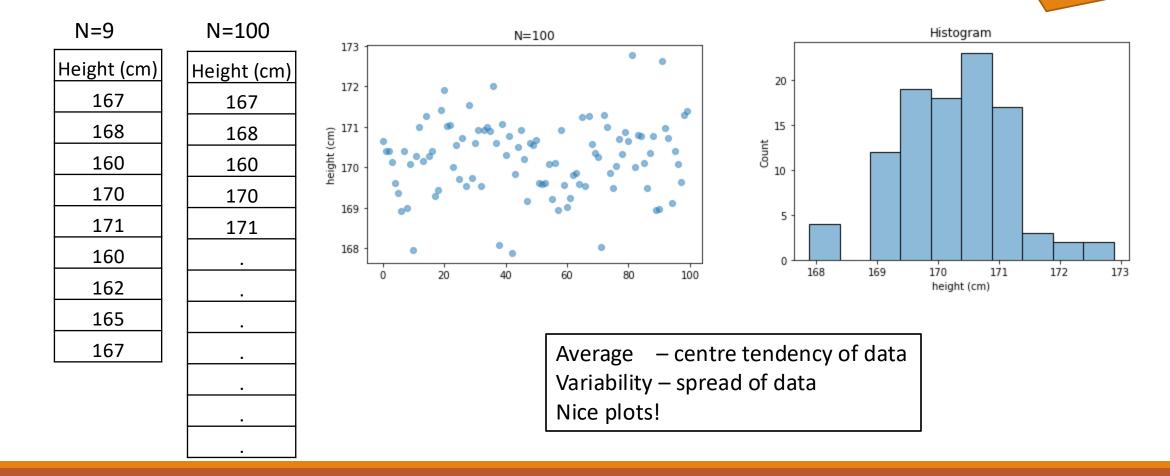
# Describe your data

### Describe your data

- Types of Variables
- Descriptive Statistics:
  - Average: Mean, Mode, Median, Frequency distribution
  - Spread/variability: Range, Percentile, Standard deviation
  - Skewness, Outliers

Visualization: Plots

### How would you describe it?

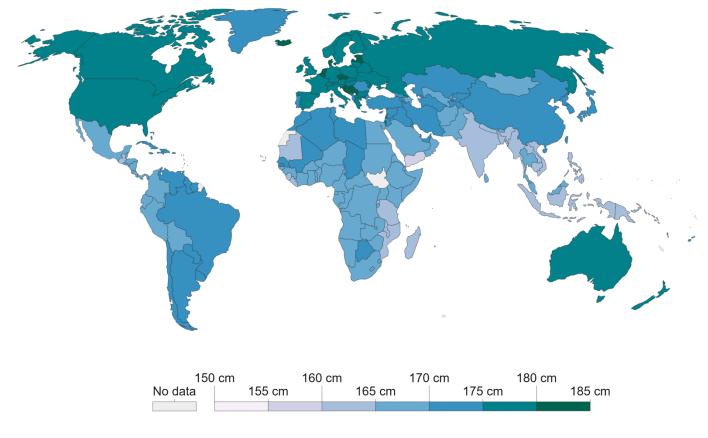


# Example:

#### Average height of men by year of birth, 1996



Mean height of adult men by year of birth. Data for the latest cohort (the year 1996) is therefore the mean height of men aged 18 in 2014.



Source: NCD RisC, Human Height (2017)

OurWorldInData.org/human-height • CC BY

#### Source: https://ourworldindata.org/grapher/average-height-of-men

Types of Variable

#### Numerical

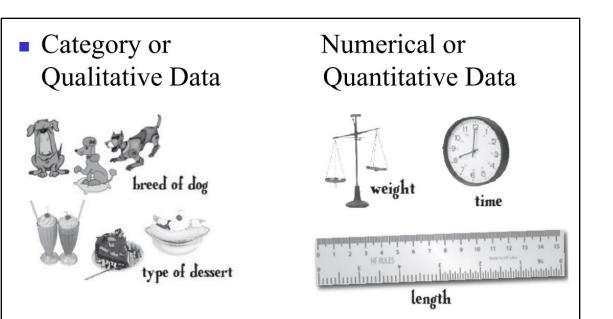
 Quantitative: blood pressure, sugar level, no of cells, height, BMI Continues, Discrete

Categorical

- Qualitative: ethnicity, disease or not? , sex? binary (2 categories), nominal (>2 cat.)
- Ordinal: satisfaction-rating, age-group

Operators (where?)

• +. -, X



# Visualisation – just plotting the data

#### Impression of visualization: Profit of company

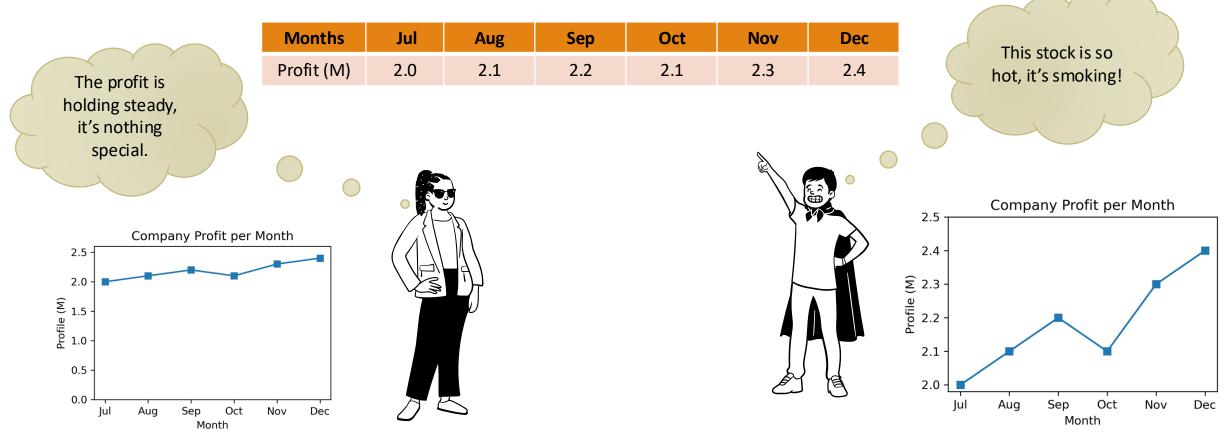


Image Ref: Book: First Head "Statistics" O'Relley

# Plot & type of variable

### Numerical

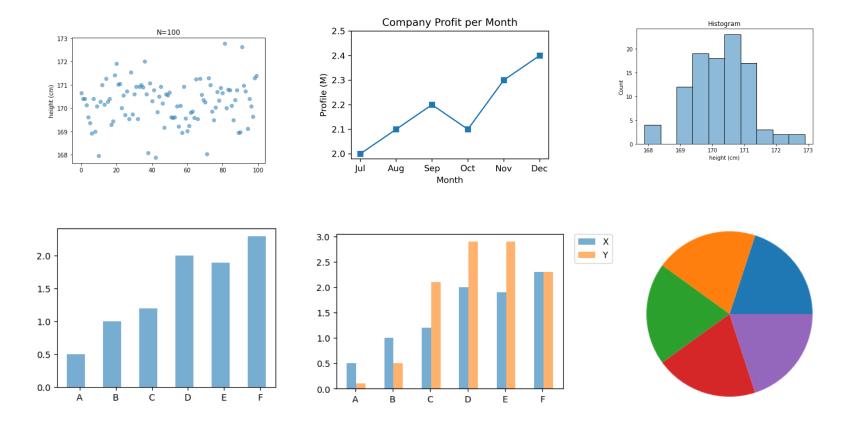
• Quantitative: (height)

### Categorical

• Qualitative:

	Genre	Unit Sold	
	Sports	27,500	
	Strategy	11,500	
	Action	6,000	
	Shooter	3,500	
	Other	1,500	
• Ordinal:			

Hours	Frequency	
01	4,300	
13	6,900	
35	2,000	
510	1,000	
1024	3,000	



#### Ref: Book: First Head "Statistics" O'Relley

### **Descriptive Statistics**

Summarizing the data

- Average: Mean, Mode, Median
- Frequency distribution
- Spread/variability: Range, Percentile, Standard deviation
- Skewness, Outliers

• What?, When?, Which?

### Average: mean, mode, median

Most representative value of data

- Height in class

[4, 4, 5, 4, 4, 4, 5, 4, 3, 5.5, 6, 4.5, 4.2, 5.2, 5,5, 6,1]

- Preference of drink

[tea, tea, coffee, coffee, tea, tea, milk, tea, coffee, coffee]

- Age-group

[10-15, 10-15, 10-15, 15-20, 20-25, 20-25, 20-25]

Mean: sum of all values/ number of values

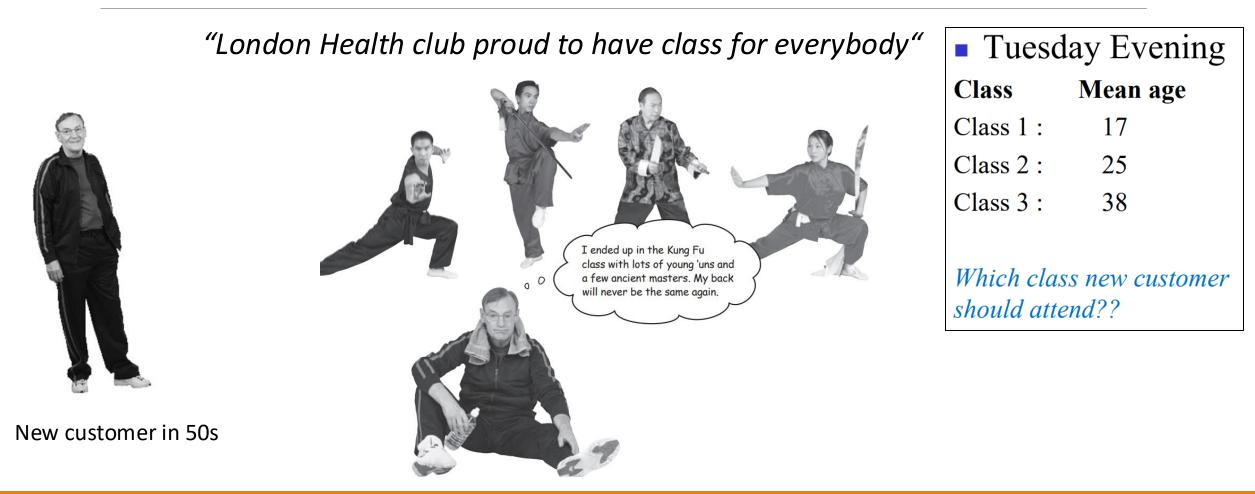
**Median**: middle value of sorted sequence

Mode : most frequent value

### Let's see a case: A Health Club



A perfect match for you



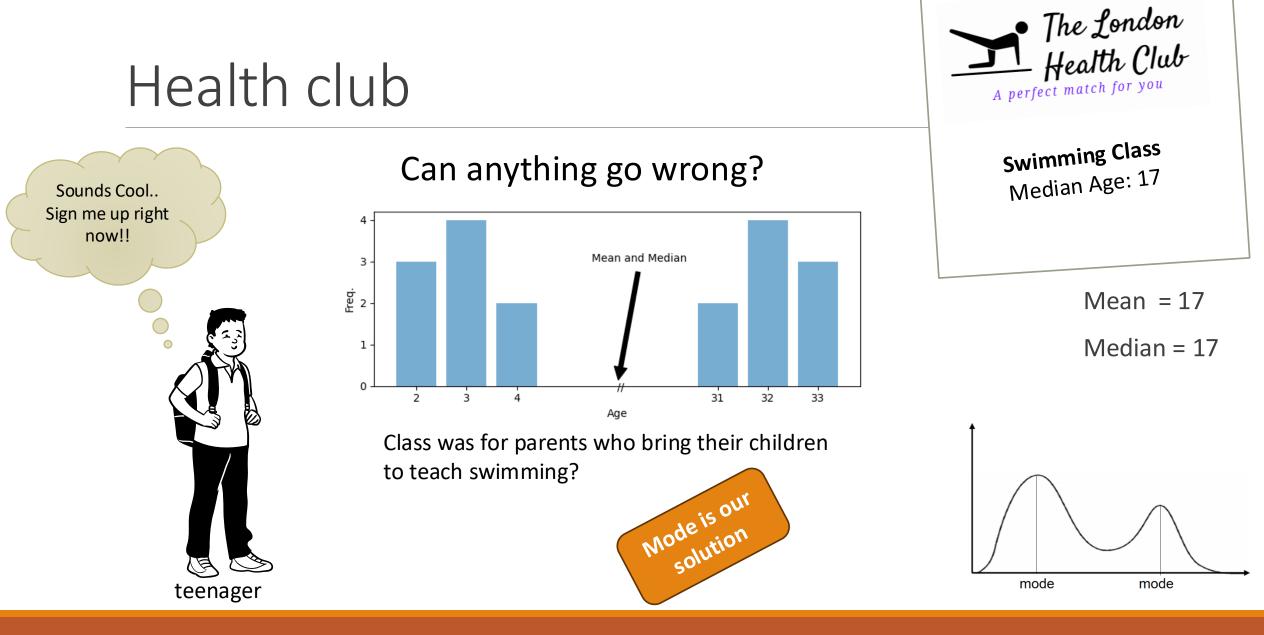
#### Ref: Book: First Head "Statistics" O'Relley



### Health club



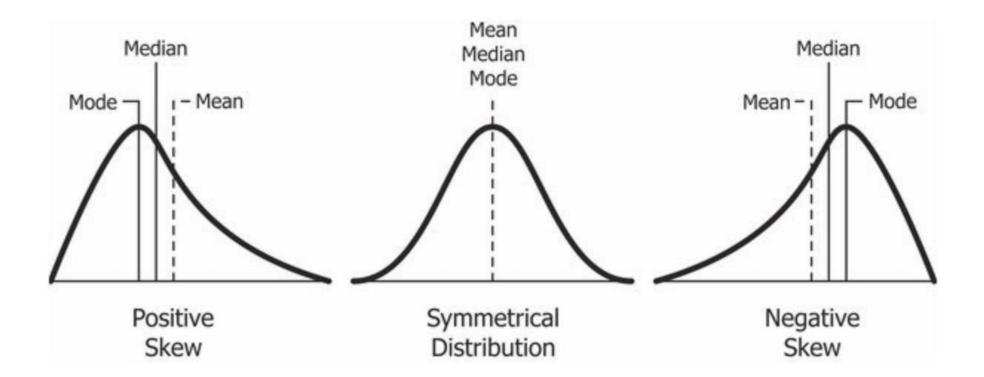
#### Ref: Book: First Head "Statistics" O'Relley



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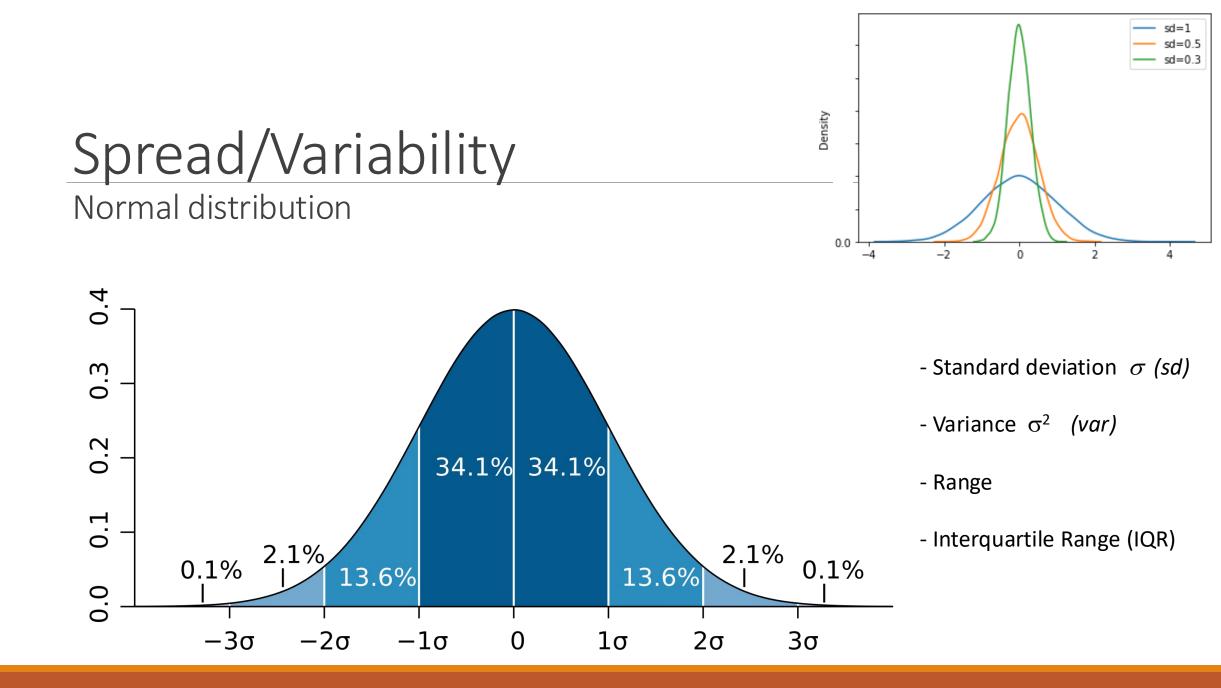
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### Skewness



#### https://en.wikipedia.org/wiki/Skewness

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https://en.wikipedia.org/wiki/Normal\_distribution

### Question

You are a coach for a cricket (or football) team, need to hire a new player

- Player 1, mean score of run rate (or goal rate) is 8, with standard deviation of 4
- Player 2, mean score of run rate (or goal rate) is 8, with standard deviation of 2

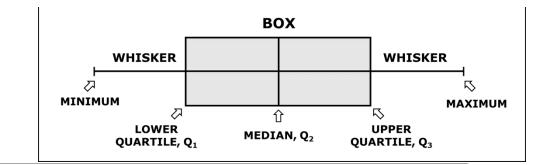
Who would you hire?



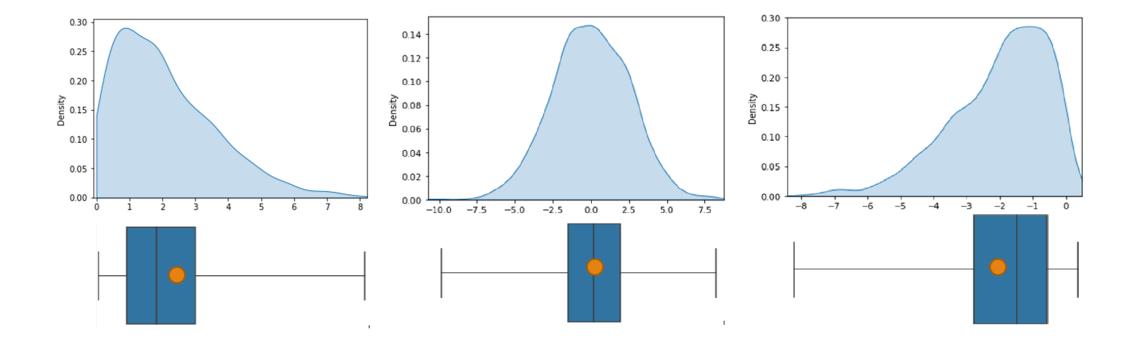


Player 1

Player 2

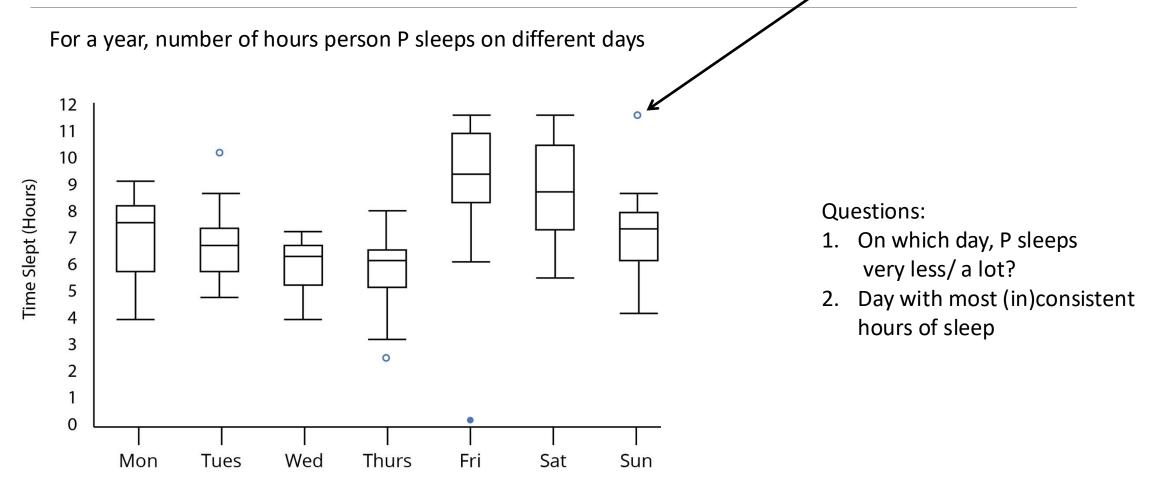


### Box-whisker plot



<u>Outlier</u>: > Q3 + 1.5xIQR < Q1 - 1.5xIQR

### Visual comparison with boxplot



### Can we categorise cont. data?

- To improve the interpretation
- Example : BMI into 2 or 3 categories high or low BMI
- Implications?
- Loss of information loss of statistical power to detect the differences
- Impact of choosing –where to cut
- Splitting at median (dichotomising) reduces statistical power
- Worst for binary than 4 or more categories

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# Introduction to Statistics

### Part 2: Inferential Statistics

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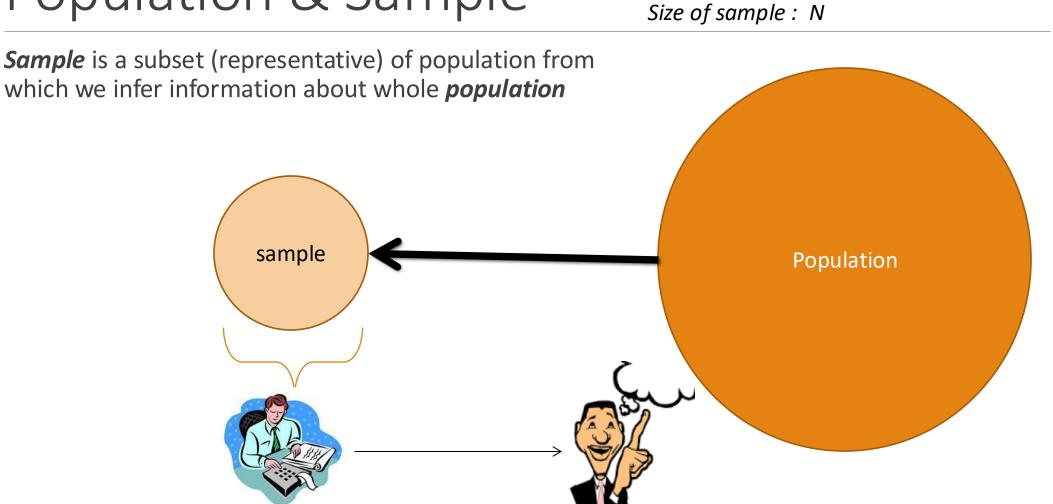
# Inferential Statistics

### Inferential Statistics

- Sample and Population
- Estimate population parameters from sample, and its accuracy (standard error)
- Standard error and standard deviation
- Confidence interval
- Size of data

*Population: A complete set of individual (objects)* 

Sample aka sample-data One point in sample-data aka item, point, sample



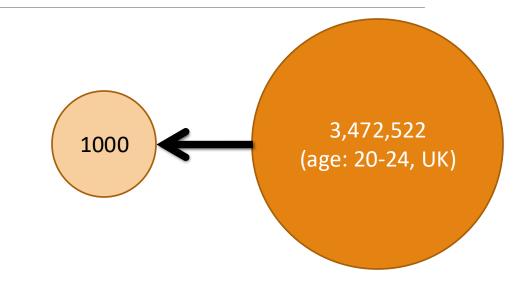
### Population & Sample

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### Examples:

- 1. Average weight of individual in UK of age 20-24.
- 2. Average marks of med. student in first year at Imperial.
- 3. Average Heart Rate of patients with High Blood Pressure in UK
- 4. Average height of Italian men

- Q: Why take sample, why not entire population?
- Q: How to sample (extract points/items from population)
- Q: How many points (samples, items) are required in a sample



### Estimation of parameter

Mean weight of population

Best guess:

- Sample mean is estimation of population mean
- Uncertainty of this estimation (not exact value)



CI for the mean weight		
Mean	95%Cl	
169.5	163.7 – 174.2	

3,472,522

(age: 20-24, UK)

1000

Plausible values for true unknown

- Confidence Interval (CI)

### Best guess: - Sample mean weight of is estimation of population mean weight - Uncertainty of this estimation (not exact value)

Estimation of parameter

Accuracy of best guess

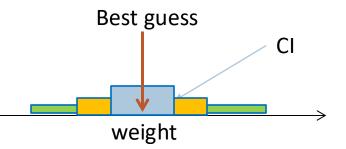
Mean weight of population

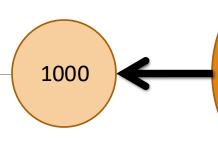
-Standard Error (SE)

Plausible values for true unknown

- Confidence Interval (CI)

	CI for the mean weight		
Mean	90%CI	95%CI	99%CI
169.5	165.5-173.4	163.7 – 174.2	163.0-175.9





3,472,522 (age: 20-24, UK)

## Standard Error and CI

Standard Error:

 $SE = \frac{SD \ of \ population}{\sqrt{sample \ size}} \approx \frac{SD \ of \ sample}{\sqrt{sample \ size}}$ 

95% Confidence Interval

95%  $CI = sample mean \pm 1.96 \times SE$ 

*Means of all (hypothetical) samples follow normal distribution and 95% of them lie within mean* ± 1.96xSE

### Standard Deviation Vs Standard Error

SD:

- measure of spread/variability of data
- descriptive statistics
- for normally distributed data, 2SD includes 95% of observed values

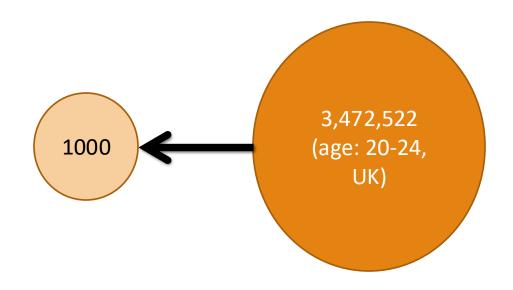
SE:

- accuracy of estimation of population
- inferential statistics
- for 95% Cl, hypothesis testing etc
- range of values likely to include true population parameter

# When it can be misleading?

- Sample is not **representative** of population (validity)

- Not large enough data (accuracy)





### Let's compute – Lab session

Data

Sample mean

Standard deviation

Standard error

Confidence interval

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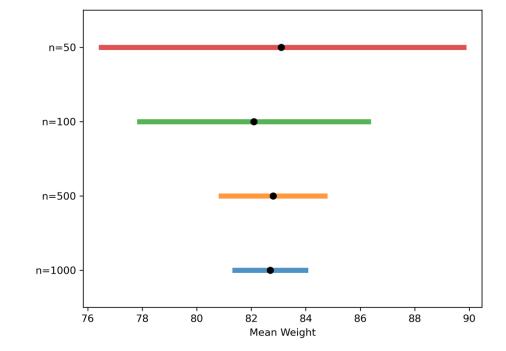
# Effect of sample size: Example 1

#### Weight:

Sample mean = 81.4 kg, Standard Deviation = 21.4kg, n=1000

SE = ?, CI= ?

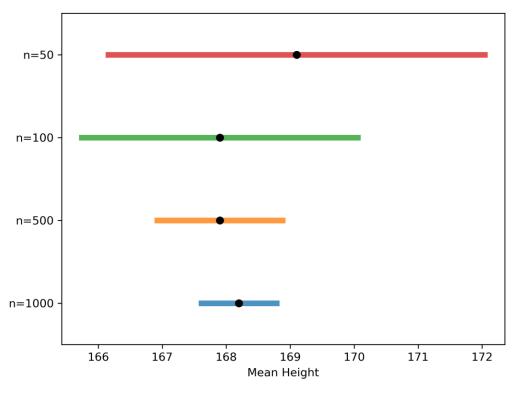
N of sample	Weight (mean)	Weight (SE)	95%Cl
50	83.1	3.3	76.5-89.8
100	82.1	2.1	77.9-86.3
500	82.8	0.96	80.9-84.7
1000	82.7	0.68	81.4-84.0



## Effect of sample size: Example 2

Height:

N	Mean	SE	CI
50	169.1	1.5	166.16-172.04
100	167.9	1.1	165.744-170.056
500	167.9	0.5	166.92-168.88
1000	168.2	0.3	167.612-168.788



## Simulation: Sampling

https://nikeshbajaj.github.io/teaching/demos/Stats/sampling

Others

https://onlinestatbook.com/stat\_sim/sampling\_dist/index.html

https://onlinestatbook.com/2/index.html

### Proportion and Cl

Estimating proportion of population that have particulate condition A.

Find the proportion in sample p = #A/total

SE of the proportion 
$$p = \sqrt{\frac{p(1-p)}{n}}$$

95%CI of the proportion  $p = p \pm 1.96 \times SE$ 

\*np>5 & n(1-p)>5

#### Example:

Find proportion of obese people (BMI>30), given sample of 1000 people, among which 391 are obese.

p = 391/1000 = 0.391 SE = 0.0154 95%Cl = 0.362 to 0.422

**Question:** 151 have asthma in 1000, compute..?

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# Introduction to Statistics

### Part 3: Hypothesis: Design & Testing

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# Given two groups of data

TEST FOR DIFFERENCES

TEST FOR ASSOCIATIONS

## Hypothesis Testing (p-value)

- Hypothesis? testing? P-value?
- Type I and type II error,
- Multiple testing and statistical power

## Hypothesis and Testing

- Hypothesis: A statement about a true value of parameters and their relationship in a defined population
- Testing: The procedure, based on sample, to determine if the hypothesis is a reasonable statement\*





## Define Hypothesis

In science to verify if a hypothesis is a reasonable statement, you need to test it against its contrary which is assumed to be true.

### Null Hypothesis H0

Assumed to be true  $\rightarrow$  No true difference or relationship between observed values in the sampled population

Alternative Hypothesis H1 { "Burden of Proof" }

To be proven  $\rightarrow$  There **IS** true difference or relationship between ..

You have a question/state Define hypothesis

## Example 1

Is lung function different between genders?

H0: lung function in men = lung function in women

H1: lung function in men  $\neq$  lung function in women

### Alternative Hypothesis with sides

- H1: lung function in men  $\neq$  lung function in women two sided
- H1: lung function in men < lung function in women one sided
- H1: lung function in men > lung function in women one sided

You have a question/state D hyp



## Example 2

Is height in a class A and class B different?

H0: ?

H1:?

### Question:

Can an alternative hypothesis be:

- There is no difference between two samples?
- Two sample groups are same?



### Question:

**Example: Let's make some hypothesis** 

- Your friend is **always** late and never agrees that he is.

Statement?:

Acquire Data

## Acquire Data

Acquiring Data:

- Conducting experiment in Lab
- Collecting data from other sources

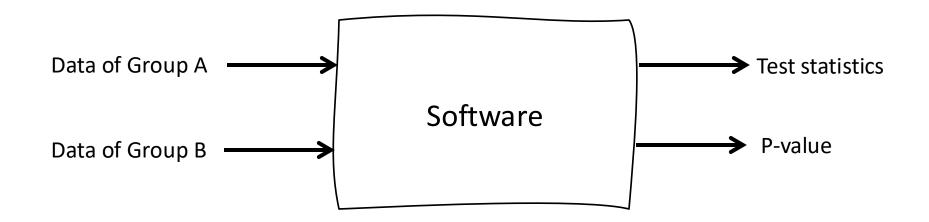
This is your sample data (think of population).

Ideally, this should be representative of population and large enough



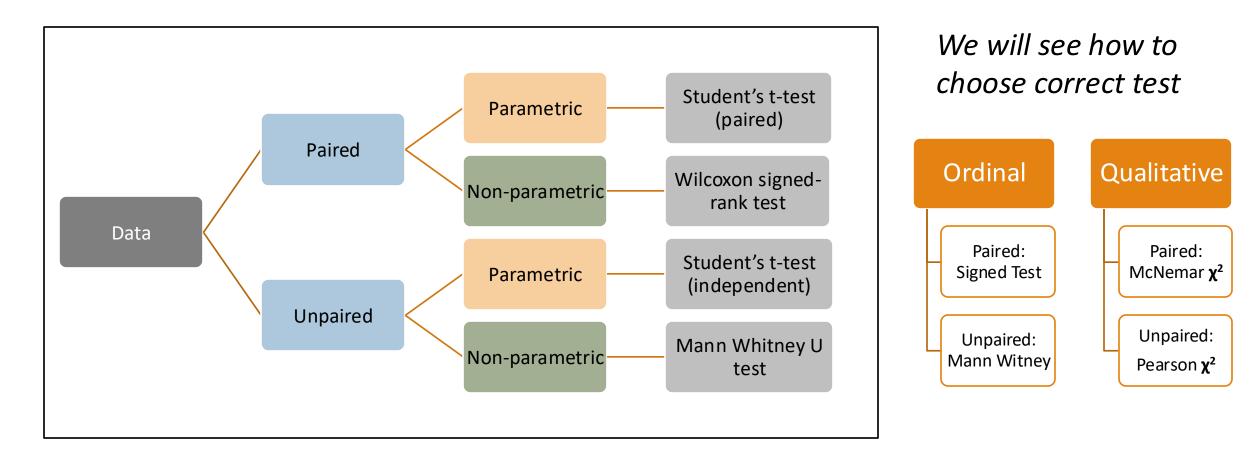
## Performing a test

Performing a test on a two groups for establishing differences or association, we use software.



**Good news!** You don't need to remember the formulas But you need to know, which test to perform and how to the read results

### Perform test Choosing a right statistical test



Perform

Test

## Test & P-value

With a right test, compute test statistics, that summarise the different/relationship in your sample.

- Use test statistics to compute p-value, that tells you either to accept or reject null hypothesis

**P-value:** Probability of obtaining the difference/effect observed in given sample by pure effect of **chance**, when null hypothesis is true.

- If two samples comes from same population, how likely we see a difference between them
- Ranges from 0 to 1.
- $\,\circ\,$  To conclude a statistical significance , we need a cut-off value  $\alpha$  (i.e. 0.05)
- NOT the probability of making a mistake!

### P-value

Example:

- If p-value is <0.05, we are confident enough to reject the null hypothesis.
- $\alpha$  = 0.05  $\rightarrow$  5% chance of rejecting null hypothesis, even if it is true.
- $\alpha$  = 0.05  $\rightarrow$  probability of committing a type I error

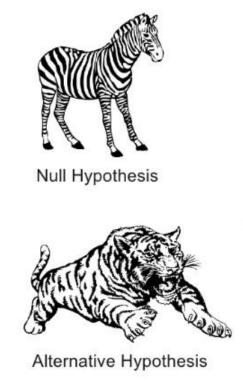
	Null hypothesis H0		
	True		
Accept H0 (fail to reject H0, $p>\alpha$ )	Right	Type 2 Error (False negative with probability $\beta$ )	
Reject H0 (p<α)	Type 1 Error (False positive with probability $\alpha$ )	Right	

https://www.slideshare.net/smulford/type-1-and-type-2-errors

### Interpret p-value

## Type 1 and Type 2 Error





https://www.slideshare.net/smulford/type-1-and-type-2-errors

## Type 1 and Type 2 Error

Interpret p-value



Null Hypothesis

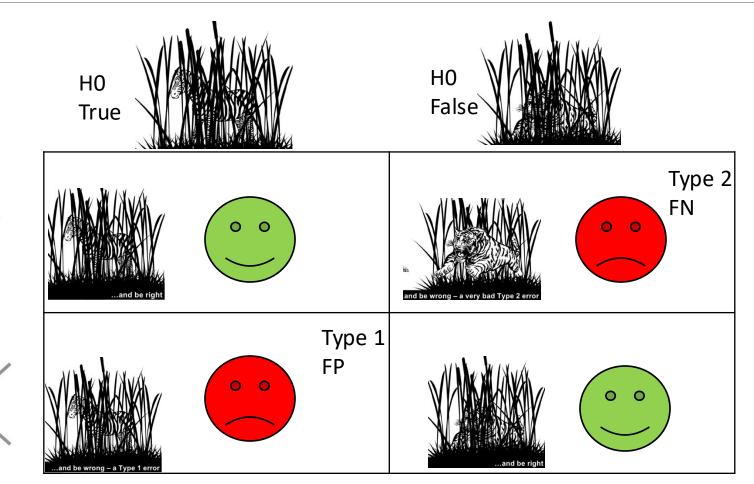


Alternative Hypothesis



Null Hypothesis

**Null Hypothesis** 



### Interpret p-value

## Type 1 and Type 2 Error

Type 1 Error ( $\alpha$ ):

- Rejecting TRUE null hypothesis
- False Positive
- $\,\circ\,$   $\alpha$ =0.05, 5% probability of false positive

### Type 2 Error ( $\beta$ )

- Failing to reject FALSE null hypothesis
- False Negative
- Power is probability that we correctly rejects the Null Hypothesis
- Power = 0.8,  $\beta$  =0.20 (1-power), power 80%

## A fun way to remember Type of Error



## P-value: summary

Smaller the p-value, stronger the evidence against null hypothesis

If p-value is <0.05:

- It is unlikely that any difference found in samples are due to chance
- Reject the null hypothesis in favour of alternative hypothesis
- Statistical significance

If p-value is <0.001:

• Strong evidence of significant results

What is p=0.049 or p=0.051?

- p-value is a guideline to decide if results deserves second look
- Ref: <u>Scientific method: Statistical errors</u>

## Multiple testing

Each test has a 5% chance of 1 false positive

So, running multiple tests increases the probability of false positive

On same dataset, testing for multiple outcomes, single outcomes in multiple sub-groups, or multiple effects.

- Before testing, limit your objectives and outcomes to be tested.
- If you have to apply for multiple testing, apply  $\alpha$  correction methods (e.g. Bonferroni,  $\alpha/n$ )

## Significance and Meaningful

Statistically significant results does not always have meaningful relevance and vice-versa

- For that, we check
- Effect Size
  - Difference in Mean
  - Cohen's D

Example:

- Two class groups A and B, have statistical significant (p<0.001) difference of 2 marks in given course.
- Men and women have a statistically significant difference of 0.5mL in lung function

## Power and Sample size (N)

- Low power (due small sample size ) increase the probability of False Negative

- You might find no difference between groups, and that might be False Negative, due to high  $\beta$  or low power (small sample size)

Example

Group	N	Mean	sd
Male	421	3555.20	909.75
Female	407	2500.77	625.99

Difference in mean = 1054.43 p-value <0.0001 T-statistics = 0.81

Group	Ν	Mean	sd
Male	12	3548.08	917.3
Female	8	2993.00	571.3

Difference in mean = 555.08 p-value = 0.4319 T-statistics = 0.81

## Sample Size calculation

For two groups with mean m1 and m2, and standard deviation of sd, we need N samples in each group to be able to reject a null hypothesis with probability of False positive as 5% and probability of False negative of 80%

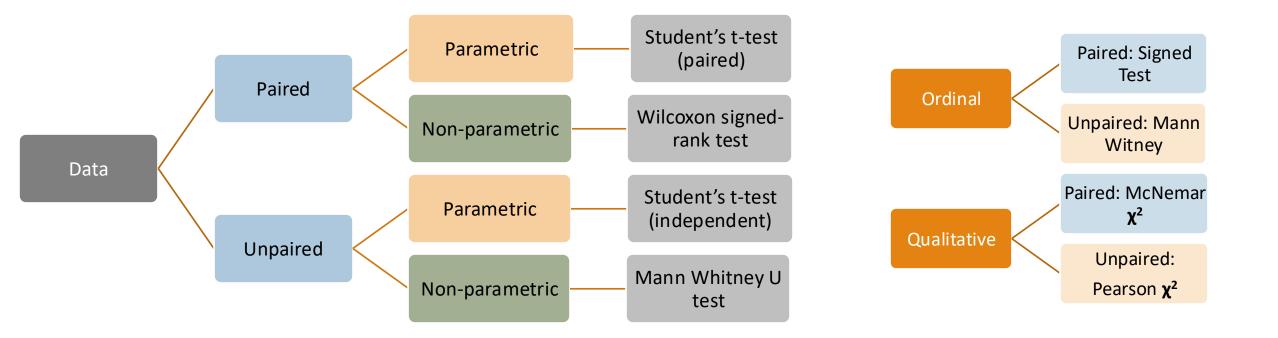
N in each group = 
$$f(\alpha, \beta) \times \frac{2(sd^2)}{(m_2 - m_1)^2}$$

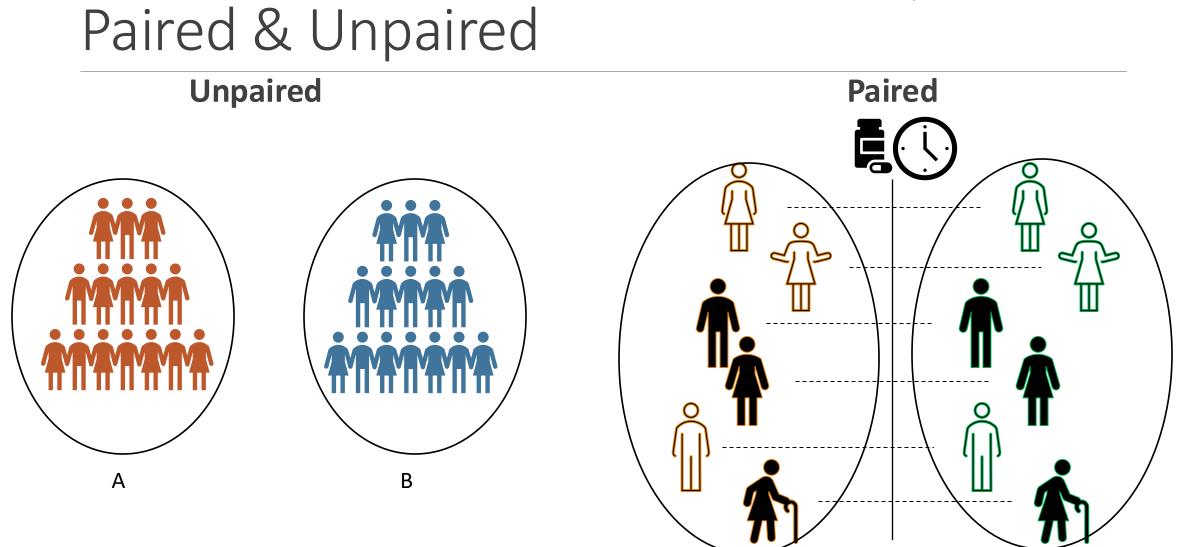
 $f(\alpha,\beta) = f(0.05,0.20) = 7.85$ 

Don't worry about the formula, it is available in all the software.

Important thing to notice  $\rightarrow$  smaller the difference you like to detect, more samples you need, smaller the sd is less sample you need

## Choosing the correct test





# Paired & Unpaired

Unpaired (independent)	Paired (dependent)
- Data collected from each sample is independents of time (usually collected once)	- Data collected from same subjects at different time (before and after treatment)
- Different subjects in different groups	- Same subjects in different groups
- BMI, age, height of subject	- BMI, before and after a treatment

## Parametric & Non-parametric

Parametric Tests

- Relies on the underlaying statistical distribution
- Normally distributed data (normality test)

Non-parametric Tests

• Do not depend on any distribution

### Normality and Homogeneity of Variance

- Normality test : Shapiro-test
- If test is not significant (p<alpha) that indicates the sampling distribution is normally distributed.

- Test for Homogeneity of Variance: Levene test
- The small p-value suggests that the populations do not have equal variances

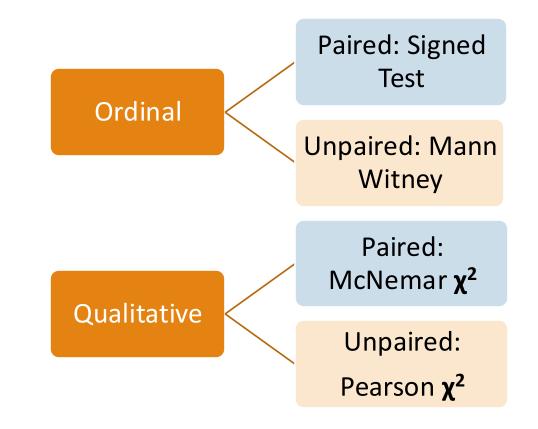
# For Categorical Data

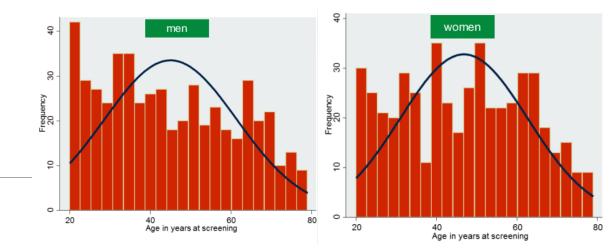
### Numerical

- Quantitative: blood pressure, sugar level, no of cells, height, BMI
  - Cont., Discrete

### Categorical

- Qualitative: ethnicity, disease or not?, sex?
  - Binary (2), nominal (>2)
- **Ordinal**: satisfaction-rating, age-group





# Group N mean age sd sd² Men 514 45 16.41 269.38 Women 486 46.83 15.87 251.81

Test statistics: 1.79

P-value: 0.0741

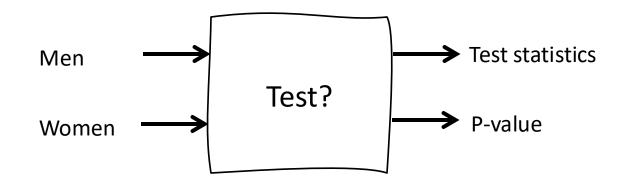
### Age difference?

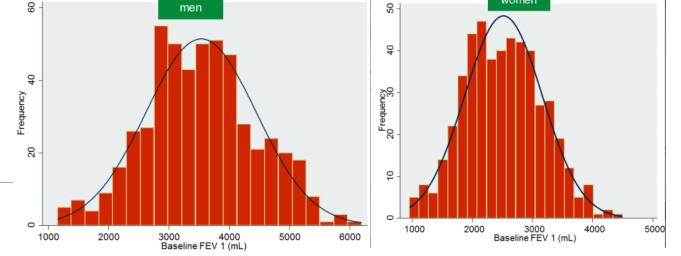
## Example 1

*Is average age of men and women different in given sample (dataset\*)?* 

- HO:  $\mu_{men} = \mu_{women}$
- H1:  $\mu_{men} \neq \mu_{women}$

### Paired? Normality? Equal variance?





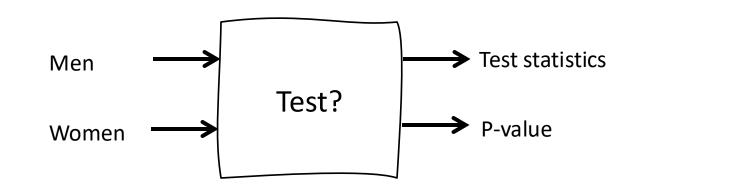
## Example 2

*Is the average lung function in men different to one in women in general population* 

- H0:  $\mu_{men} = \mu_{women}$
- H1:  $\mu_{men} \neq \mu_{women}$

Group	Ν	mean fev1(mL)	s d	
Men	514	3535.08	915.08	
Women	486	2515.17	646.08	

### Paired? Normality? Equal variance?



P-value: 0.001

Test statistics: 20.26

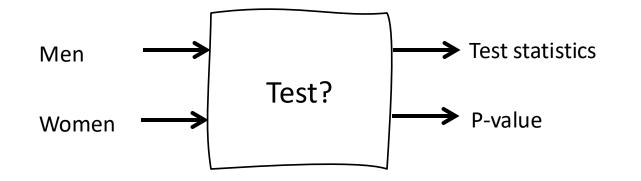
### difference?

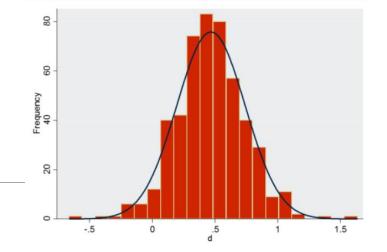
## Example 3

Has the lung function changed after intense exercise in the study

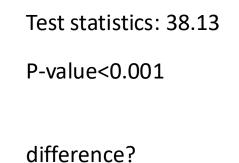
H0 
$$\mu_{before} = \mu_{after}: \mu_d = 0$$
  
H1:  $\mu_{before} \neq \mu_{after}: \mu_d \neq 0$ 

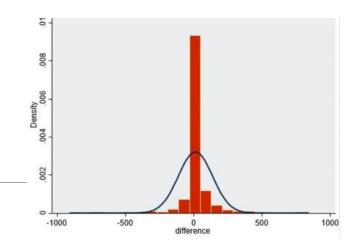
### Paired? Normality? Equal variance?





	N	mean	median	sd	sd
d	496	0.47	0.47	0.27	0.01



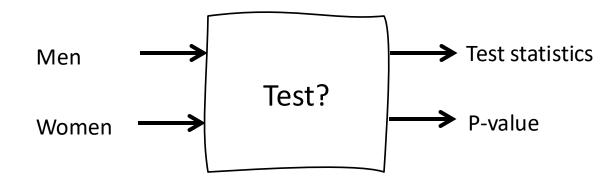


## Example 4

Has total immunoglobulin E (IgE) changed over time (over 10 years)

H0 
$$\mu_{before} = \mu_{after}: \mu_d = 0$$
  
.  
H1:  $\mu_{before} \neq \mu_{after}: \mu_d \neq 0$ 

### Paired? Normality? Equal variance?



Test statistics: 10 P-value>0.05

## Let's give it a try

From the statement from list, let's define a hypothesis, testing approach:

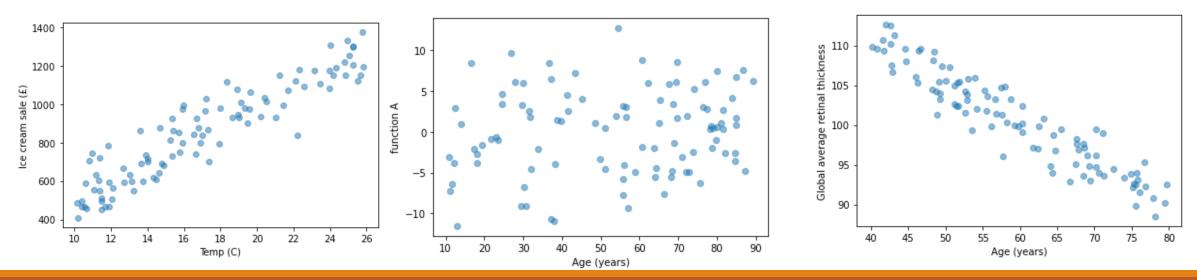
- 1. Effectiveness of a teaching method for a subject
- 2. Effectiveness of a drug on elderly >50 for lung function
- 3. Lung function of smokers and non-smokers

## Association between two

### Correlation

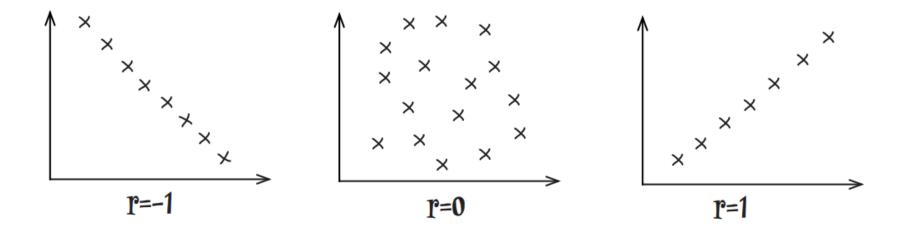
- Investigate a relationship between two independent variables (i.e. x and y)
- Does x increase as y or vice-versa?
- Is relation linear?

One simple way is to plot scatter graph and see

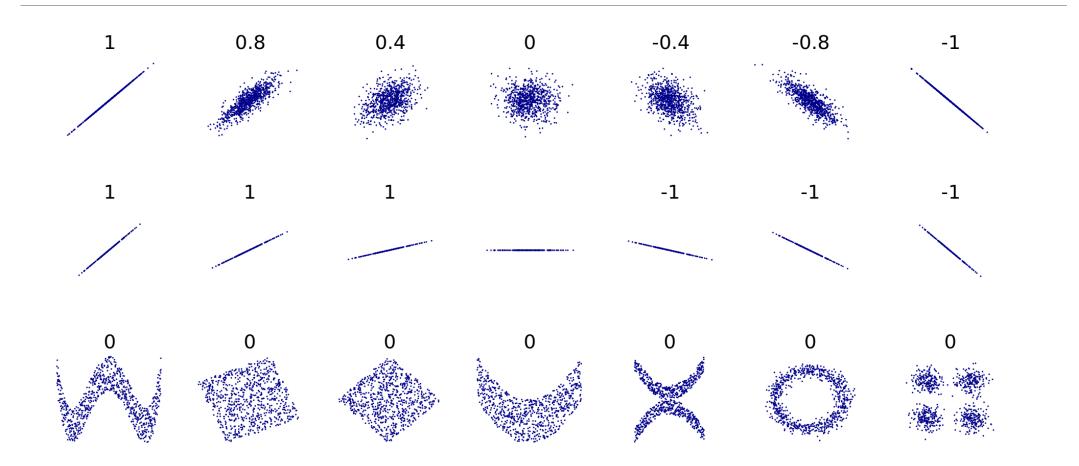


## Quantifying Correlation

Pearson Correlation Coefficient r or  $\rho$  (rho)



### Pearson Correlation Coefficient



### https://en.wikipedia.org/wiki/Correlation

## Correlation

### **Pearson Correlation Coefficient**

### Parametric test

- x, y: normally distributed
- linear relationship

### Generally:

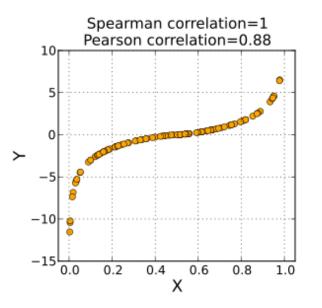
- $|r| < 0.4 \rightarrow$  weak
- 0.4 <  $|r| < 0.7 \rightarrow$  moderate
- $0.7 < |r| \rightarrow \text{strong}$

### r = 0, no *linear* relationship

### **Spearman Rank correlation**

### Non-parametric test

Based on ranks rather than exact values



## Correlation and P-value

P-value can be obtained from correlation with

Null Hypothesis H0 : r = 0

Alternative Hypothesis  $H1: r \neq 0$ 

P-value tells us the probability of getting high correlation between x and y by pure chance

### Examples

- 1. BMI vs Age, r= 0.13, p-value =0.08
- 2. BMI vs Age, r= 0.13, p-value =0.04
- 3. lung function before vs after exercise, r= 0.93, p-value = 0.001

### Correlation

A strong correlation between x and y does **Not mean** 

- x causes y :  $X \rightarrow Y$
- y causes x :  $Y \rightarrow X$

• x and y are caused by one or more other variables z:  $Z \rightarrow X$ ,  $Z \rightarrow Y$ 

### Correlation is not causation

## For single or more than 2 samples

### One Sample t-Test

- Also known as single sample t-test
- When you have only one sample, to test if the mean of sample is equal to some well stablished/ hypothesized mean
- Null hypothesis H0 :  $\mu_s = P$
- Alternative hypothesis  $H1: \mu_s \neq P$

#### ANOVA

- Analysis of Variance, generalised for more than 2 groups
- Tests whether mean of all the groups are similar or not
- Null hypothesis: There is no difference in means between groups
- Alternative hypothesis: At least one group mean is significantly different from the overall mean

Stats Demo links:

https://nikeshbajaj.github.io/teaching/demos/Stats/sampling https://nikeshbajaj.github.io/teaching/demos/Stats/hypothesis

Resources/Video/Material:

https://nikeshbajaj.github.io/teaching/stats

If you have any question or doubt, please contact me via email

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